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Neural-Network-Augmented Empirical Interpolation Method for Field Reconstruction with Noise and Vibration Tolerance

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Abstract. Field reconstruction methods based on machine learning often face challenges when handling unstructured data and designing complex model structures. On the other hand, Reduced Order Models (ROM) struggle with robustness in the presence of observation noise and sensor vibrations. Striking a balance between lightweight design, the ability to handle unstructured data, and robustness presents a significant challenge. In this paper, we introduce the EIM-NN algorithm, which leverages neural networks to determine the coefficients of the subspace found by the EIM algorithm. Furthermore, we present the EIM-TNN algorithm, which enhances robustness by designing a loss function incorporating Tikhonov regularization. Our neural network consists of only two fully connected layers, allowing it to handle unstructured data while maintaining a lightweight profile. Experimental results demonstrate the algorithm's ability to significantly enhance robustness against noise and sensor vibrations without compromising its accuracy in fitting the original data. Additionally, the algorithm's lightweight nature ensures that the added training time and memory requirements over EIM remain acceptable, making it adaptable to a range of industrial applications.

AMS subject classifications: 68Txx

Key words: Field reconstruction, EIM-TNN, lightweight, observation noise, sensor vibration.

1 Introduction

In the past, the primary approach to field reconstruction involved the utilization of numerical optimization methods, particularly by developing computationally efficient

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reduced-order models (ROMs) [1–3]. Among the various ROM techniques, Reduced Basis (RB) methods [4–6] have established themselves as a well-established and widely adopted class of ROM techniques. RB methods typically revolve around identifying a set of reduced basis functions within a lower-dimensional subspace. Subsequently, they map low-dimensional observations to high-dimensional physical fields by determining the coefficients of these basis functions.

Different approaches can be employed to construct the reduced basis, including greedy algorithms [7, 8] and Proper Orthogonal Decomposition (POD) [9, 10]. Additionally, the Empirical Interpolation Method (EIM) [11–13] represents a non-intrusive ROM technique that constructs an empirical interpolation from the solutions of the parametrized mathematical model or from observations of the physical state to be approximated. EIM and its generalized version, GEIM [14–17], enable the integration of the mathematical model and observation data to reconstruct full order fields.

However, in real industrial environments, sensor observations are often subject to contamination, which can stem from errors in observation readings, vibrations in the sensor's position, and other sources. In the presence of contaminated observations, the performance of EIM and GEIM is compromised, as they may no longer converge and can even exacerbate observation noise [18]. It is important to note that these ROMs often entail solving complex optimization problems, demanding a substantial number of iterations and, consequently, leading to high computational requirements.

In recent years, there has been a notable upward trend in the integration of machine learning techniques into field reconstruction endeavors. This trend aims to enhance both computational efficiency and robustness, capitalizing on the nonlinear capabilities of neural networks [19–23]. For instance, in a specific case, Fukami et al. [24] utilized Voronoi tessellation to construct a structured grid representation based on sensor positions. Subsequently, they employed Convolutional Neural Networks (CNNs) to establish a mapping from mobile sensors to the physical field. In a related vein, Gong et al. [25] proposed the application of the VCNN model to nuclear reactors to address the observation inaccuracy arising from sensor position vibrations due to reactor aging. Building upon these foundations, Li et al. [26] took further strides in optimizing their machine-learning model. They achieved this by amalgamating the Voronoi tessellation method with the Residual neural network [27] and deliberately incorporating sensor vibration and observation noise into the training data. This innovative approach led to the development of a Noise and Vibration Tolerant ResNet (NVT-ResNet), which was specifically designed to enhance the robustness of the reconstruction model in the presence of noise and sensor vibrations. This fusion approach resulted in improved robustness for the reconstruction model against noise and sensor vibrations, leading to enhanced reconstruction accuracy.

While machine learning models offer robust and efficient solutions [28], it is essential to acknowledge that ROMs grounded in physical principles possess distinct advantages. These advantages include generalizability, interpretability, and adherence to well-established physical principles [29]. As a result, there has been a concerted effort in recent research to introduce nonlinear operators into the low-dimensional space within ROMs.