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# Solving Multi-Group Neutron Diffusion Eigenvalue Problem with Decoupling Residual Loss Function

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**Abstract.** In the midst of the neural network's success in solving partial differential equations, tackling eigenvalue problems using neural networks remains a challenging task. However, the Physics Constrained-General Inverse Power Method Neural Network (PC-GIPMNN) approach was proposed and successfully applied to solve the single-group critical problems in reactor physics. This paper aims to solve critical problems in multi-group scenarios and in more complex geometries. Hence, inspired by the merits of traditional source iterative method, which can overcome the ill-condition of the right side of the equations effectively and solve the multi-group problem effectively, we propose two residual loss function called Decoupling Residual loss function and Direct Iterative loss function. Our loss function can deal with multi-group eigenvalue problem, and also single-group eigenvalue problem. Using the new residual loss functions, our study solves one-dimensional, two-dimensional, and three-dimensional multi-group problems in nuclear reactor physics without prior data. In numerical experiments, our approach demonstrates superior generalization capabilities compared to previous work.

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**Key words**: Deep learning, eigenvalue problem, nuclear reactors, multi-group problem.

## 1 Introduction

The neutron diffusion equations are fundamental equations in nuclear reactor physics, which are used to describe the transport behavior of neutrons in a nuclear reactor and

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derived by neutron diffusion theory [1]. Depending on different physical situations and assumptions, the neutron diffusion equations can be further developed into more complex equations, such as the multi-group diffusion equations, neutron noise equations, and so on. The neutron diffusion equation can be simplified from the Boltzmann transport equation, which accurately describes the neutron transport process, and the neutron diffusion equation includes single-group, multi-group, transient and steady-state problems. The neutron diffusion equations can be used for fuel management to determine the optimal arrangement of fuel rods, thereby maximizing fuel utilization. The neutron diffusion equations can also be used for reactor operation control to maintain the stability and safety performance of the reactor, such as controlling the power of the reactor by controlling the neutron flux in the reactor.

Over the past few decades, researchers have developed a range of solution methods for problems such as finite difference [2], finite element [3], finite volume [4, 5], nodal expansion [6] and methods of characteristic [7], among others. However, with the potential of neural networks being explored in various fields, there is an urgent need for research in using neural networks to solve physically relevant multi-group neutron diffusion eigenvalue problems (NDEPs) in nuclear reactor scenarios.

As early as 1994, Dissanayake et al. [8] attempted to use neural network methods to solve simple cases of linear and nonlinear problems. In 1998, Lagris et al. [9] presented a more comprehensive algorithmic framework for using neural networks to solve partial differential equations. Numerous researchers have made continuous efforts in the development of related works. Currently, the most popular framework is the Physics-Informed Neural Networks (PINNs) proposed by Raissi et al. [10] This framework directly links neural networks with the information of physical equations through the form of a loss function. It is evident that neural network based methods for solving partial differential equations offer several advantages:

- Independence from mesh generation: Neural networks do not rely on mesh files for solving PDEs. Instead, they utilize collected training samples or data points, which eliminates the need for grid generation and adapts well to irregular or complex geometries.

- Integration of observational data: Neural networks have the capability to incorporate observational or experimental data into the learning process. This allows them to effectively combine the physical information from the equations with the available data, resulting in enhanced accuracy and predictive capabilities compared to traditional solver algorithms that solely rely on the physics of the equations.

- Handling high-dimensional problems: Neural networks demonstrate their advantage in dealing with high-dimensional problems, overcoming the curse of dimensionality. Neural networks can effectively learn and represent complex relationships in high-dimensional spaces, making them suitable for a wide range of problems with multiple variables or parameters.