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Scale-Decomposed Physics-Informed Neural Networks for Singular Perturbation Problems

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Abstract. A novel physics-informed neural network (PINN), scale-decomposed PINN (SD-PINN), is proposed for singular perturbation problems, such as boundary-layer flows. Singular perturbation problems exhibit sharp changes in solutions at different scales, owing to large gradients in equations, which presents a challenge to a conventional PINN. In SD-PINN, the solutions at different scales are represented by different neural networks, and their matching is achieved by imposed matching conditions. In comparison with the conventional procedure for singular perturbation problems, no domain decomposition is used. A nonlinear stretching transformation is introduced to prevent the occurrence of semi-infinite regions in the neural networks. Six benchmark singular perturbation problems are used to evaluate SD-PINN, including linear and nonlinear second-order ordinary differential equations, two-dimensional hyperbolic problems, and a flat-plate boundary-layer flow. It is demonstrated that SD-PINN can reproduce asymptotic and non-asymptotic solutions for small and finite perturbation parameters, respectively. Application of SD-PINN to the inference of perturbation parameters from available data is also discussed. Codes are available at `https://github.com/LeiZhang-code/SD-PINN`.

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1 Introduction

Boundary-layer problems are pivotal in aerodynamics and hydrodynamics [1, 2]. Solutions to boundary-layer flows provide essential near-wall information such as surface

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friction and pressure, which are crucial for drag reduction and noise control [3]. However, the solution of boundary-layer problems in engineering through either analytical methods or numerical simulations remains a challenge [4, 5]. These problems typically exhibit features such as sharp changes in dependent variables in thin regions [6, 7], and can be considered as singular perturbation problems across multiple scales, in which the boundary-layer scales associated with sharp changes are very different from the outer-region scales associated with the behavior of dependent variables outside the boundary layer. The objective of the present work is to develop a method for dealing with singular perturbation problems using physics-informed neural networks (PINNs) [8].

PINNs are a type of machine-learning numerical method with great potential for solving a wide variety of physical problems and the corresponding inverse problems. They have already been successfully applied, for example, to the solution of partial differential equations (PDEs) with or without missing physics [8–12], the reconstruction of full-field information from limited data [13], the extraction of physical information from experimental data [14], and the inference of equation parameters [8]. Compared with classical numerical methods, PINN and its variants have the following advantages [9]: (1) they provide a general framework to solve PDEs without requiring mesh generation and specific schemes; (2) they are efficient in solving high-dimensional problems; and (3) they can handle inverse problems. Recently, domain decomposition has been introduced to PINNs, as, for instance, in conservative PINN (cPINN) [15] and extended PINN (XPINN) [16], which employs different neural networks for distinct regimes in different subdomains. However, these domain-decomposition-based methods have to handle the interface conditions across the subdomains to guarantee continuity. Perturbation-theory-guided PINN methods, which combine PINNs with classical perturbation methods, have been developed for singular perturbation problems [17, 18]. These methods can address typical singular perturbation problems involving boundary layers. In numerical implementations, an approximate form of Prandtl's matching principle is adopted, where the interface conditions of the boundary layer must match the outer solution in the outer region, with the range of the boundary layer being specified a priori. These methods are particularly effective for small (but not finite) perturbation parameters in the asymptotic limit. A PINN with variable linear transformation has recently been proposed for thin-layer flow problems, in which the parameters of the transformation must be determined a priori [19].

This paper proposes a scale-decomposed PINN (SD-PINN) for singular perturbation problems, based on scale decomposition and nonlinear stretching transformations. The solution is decomposed into three components: an inner solution for the boundary-layer scales, an outer solution for the outer-region scales, and a matching term. Notably, there is no domain decomposition, that is, the inner and outer solutions are both valid across the entire computational domain. The governing equation is accordingly decomposed into an inner equation and an outer equation. Some methods have been proposed in which multiple scales have been adopted as inputs to solve singularly perturbed differential equations, such as tailored PINN (TPINN) [20], the component Fourier neural