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Majority-Vote on Undirected Barabási-Albert Networks

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Abstract. On Barabási-Albert networks with z neighbours selected by each added site, the Ising model was seen to show a spontaneous magnetisation. This spontaneous magnetisation was found below a critical temperature which increases logarithmically with system size. On these networks the majority-vote model with noise is now studied through Monte Carlo simulations. However, in this model, the order-disorder phase transition of the order parameter is well defined in this system and this was not found to increase logarithmically with system size. We calculate the value of the critical noise parameter q_c for several values of connectivity z of the undirected Barabási-Albert network. The critical exponentes β/ν, γ/ν and $1/\nu$ were also calculated for several values of z.

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Key words: Monte Carlo simulation, vote, networks, nonequilibrium.

1 Introduction

It has been argued that nonequilibrium stochastic spin systems on regular square lattice with up-down symmetry fall in the universality class of the equilibrium Ising model [1]. This conjecture was found in several models that do not obey detailed balance [2–4]. Campos *et. al*. [5] investigated the majority-vote model on small-world network by rewiring the two-dimensional square lattice. These small-world networks, aside from presenting quenched disorder, also possess long-range interactions. They found that the critical exponents γ/ν and β/ν are different from the Ising model and depend on the rewiring probability. However, it was not evident whether the exponent change was due to the disordered nature of the network or due to the presence of long-range interactions. Lima *et. al*. [6] studied the majority-vote model on Voronoi-Delaunay random lattices

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with periodic boundary conditions. These lattices posses natural quenched disorder in their conecctions. They showed that the presence of quenched connectivity disorder is enough to alter the exponents β/ν and γ/ν and therefore is a relevant term to such non-equilibrium phase-transition. Sumour and Shabat [7,8] investigated the Ising models on the directed Barabási-Albert networks [9] with the usual Glauber dynamics. No spontaneous magnetisation was found, in contrast to the case of undirected Barabási-Albert networks [10–12] where a spontaneous magnetisation was found lower a critical temperature which increases logarithmically with system size. Lima and Stauffer [13] simulated directed square, cubic and hypercubic lattices in two to five dimensions with heat bath dynamics in order to separate the network effects form the effects of directedness. They also compared different spin flip algorithms, including cluster flips [15], for the Ising-Barabási-Albert networks. They found a freezing-in of the magnetisation similar to [7,8], following an Arrhenius law at least in low dimensions. This lack of a spontaneous magnetisation (in the usual sense) is consistent with the fact that if on a directed lattice a spin S_j influences a spin S_i, then the spin S_i in turn does not influence S_j, and there may be no well-defined total energy. Thus, they showed that for the same scale-free networks, different algorithms give different results. More recently, Lima [14] investigated the majority-vote model on the directed Barabási-Albert network and calculated the β/ν, γ/ν and $1/\nu$ exponents. These results are different from those obtained using the Ising model and depend on the values of connectivity z in the directed Barabási-Albert network. In this work, we calculate the same β/ν, γ/ν and $1/\nu$ exponents for the majority-vote model on *undirected* Barabási-Albert network. Numerical results from the Monte Carlo simulations will be reported and discussed.

2 Model and simulation

We consider the majority-vote model, on directed Barabási-Albert Networks, defined [6, 16–18] by a set of "voters" or spins variables σ taking the values $+1$ or -1, situated on every site of an undirected Barabási-Albert Network with N sites, and evolving in time by single spin-flip like dynamics with a probability w_i given by

$$w_i(\sigma)=\frac{1}{2}\left[1-(1-2q)\sigma_i S\left(\sum_{\delta=1}^{k_i}\sigma_{i+\delta}\right)\right], \tag{2.1}$$

where $S(x)$ is the sign ± 1 of x if $x\neq 0$, $S(x)=0$ if $x=0$, and the sum runs over all nearest neighbours of σ_i. In this network, each new site added to the network selects old sites as neighbours influencing it; and the newly added spin does not influence these neighbours. The control parameter q plays the role of the temperature in the equilibrium systems and measures the probability of aligning antiparallel to the majority of neighbours.

To study the critical behavior of the model we define the variable $m=\sum_{i=1}^{N}\sigma_i/N$. In particular, we are interested in the magnetisation, the susceptibility and the reduced