

## Numerical Method for Boltzmann Equation with Soroban-Grid CIP Method

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Received 6 October 2006; Accepted (in revised version) 25 November 2006

Available online 29 January 2007

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**Abstract.** A new numerical scheme to solve the Boltzmann equation in phase space for rarefied gas is described on the basis of the Cubic Interpolated Propagation (CIP) method. The CIP procedure is extended to adaptive unstructured grid system by the Soroban grid. The grid points in velocity space can move dynamically following the spread of velocity space in a spatially localized manner. Such adaptively moving points in velocity space are similar to the particle codes but can provide higher-order-accurate solutions. Numerical solutions obtained by the Soroban-grid CIP are examined and the validity is discussed.

**PACS (2006):** 02.70.-c, 47.11.-j, 47.45.-n

**Key words:** CIP, Boltzmann equation, Soroban grid.

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### 1 Introduction

Various kinds of numerical methods have been used for the Boltzmann equation in application to plasma physics, hydrodynamics including rarefied gas, free molecules flow and so on. There are some characteristic phenomena in rarefied gas such as Knudsen layer [1], shock wave [2,3], Rayleigh problem [4]. Numerical methods for the Boltzmann equation can be roughly divided into two classes. One is called the Lagrangian methods. Among these Lagrangian schemes, one of the most popular particle methods is the Particle in cell (PIC) method [5]. This method has been considered to be quite stable even if only a few computational particles per one grid cell are used. However, this scheme essentially involves some disadvantages stemming from statistical numerical noise.

Another Lagrangian approach is the Monte Carlo method (DSMC:Direct Simulation with Monte-Carlo method) [3,6]. It has been used for rarefied gas dynamics described by

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the Boltzmann equation. These Lagrange-based schemes are stable even with the small number of particles and they have been used for some simulations of hyper-dimensional Boltzmann-Vlasov equation. However, there is still a disadvantage that the computation cost in the Monte-Carlo method is quite large for some flows like near continuum flows even though the drastic improvement of computer technology is making the computation much faster than ever before.

An alternative to the particle method or the Monte-Carlo method is the Eulerian method that uses a hyper-dimensional computational mesh in phase space. There have been some simulations of the standard Boltzmann equations performed by finite difference method [7]. We here propose to use the Cubic Interpolated Propagation (CIP) method that was proposed by Yabe [8–10]. In the CIP scheme, first spatial derivatives are introduced as free parameters on each grid point and we do not have to solve matrix to make cubic-interpolation function even in multi-dimensional cases. Therefore, the time evolution of the derivatives as well as the function values are calculated from a model equation that is consistent with the master equation and the scheme becomes the third-order accuracy in time and space. One of the biggest advantages of the CIP method is that the phase error and amplification factor are better than those of the other conventional schemes with less number of grid points [11]. This implies that the CIP method has possibility to overcome some problems that are intrinsic in the particle method or the Monte-Carlo method.

The CIP method has been successfully applied to various complex fluid flow problems of both compressible and incompressible flow, such as laser-induced evaporation, skimmer [12], bubble collapse, magnetohydrodynamics [13] and so on [14]. Furthermore, we have already succeeded in applying the CIP method to the Fokker Planck equation for plasma physics [15]. Nakamura and Yabe established the hyper-dimensional Vlasov-Poisson equation solver based on the CIP method, and Kondoh studied interaction between femtosecond-laser and matter both in microscopic and macroscopic levels using the CIP method [16].

During the development of the CIP method, the Soroban grid was proposed to achieve local mesh refinement (LMR) keeping higher-order accuracy by Yabe et. al [17]. The Soroban grid consists of lines and grid points in two dimensions, but the extension of Soroban grid to multidimensions such as six-dimensional Boltzmann equation is straightforward.

There are several merits to use the Soroban grid in the Boltzmann equation. (1) The range of velocity space can vary in time and space in accordance with the velocity space spread by the heating or acceleration. The particle code has great advantage in this respect since the particle can have its own velocity without velocity grid. Since the Soroban grid points in the velocity space can move independently from spatial grid points, the flexibility to trace such change in velocity space will be attained like in the particle code. (2) The local refinement is inevitable to treat the shock wave and other discontinuities. The Soroban grid can concentrate the grid to such discontinuities independently from the velocity space. (3) Even with such arbitrary grid points, higher order accuracy is well