Commun. Comput. Phys.
doi: 10.4208/cicp.090909.180210a

Vol. 8, No. 4, pp. 866-876
October 2010

Analysis of Effectiveness on Monte Carlo Importance Biased Transport Calculation

Haiyan Xu*, Zhengfeng Huang and Shaohui Cai

Institute of Applied Physics and Computational Mathematics, Beijing 100094, China.

Received 9 September 2009; Accepted (in revised version) 18 February 2010

Communicated by Xueqiao Xu

Available online 17 May 2010

Abstract. To obtain fine distributing calculation about the time, space and energy of neutron flux for a kind of non-stationary particle transport problem, the scheme of global Monte Carlo variance reduction is developed. In order to provide the foundation for this scheme, it is necessary to analyze its effectiveness before putting it to use. This paper fulfills this through analyzing the effectiveness of its core which is Monte Carlo transport importance biased calculation. By decomposing the arithmetic and calculating the representative objectives, the effectiveness of the Monte Carlo transport importance biased calculation is demonstrated.

AMS subject classifications: 11K45, 62D05, 82C70, 82D75

PACS: 02.70.Ss, 02.70.Tt, 02.70.Uu

Key words: Monte Carlo, importance biased transport, direct bias, indirect bias.

---

1 Introduction

It is important to obtain the fine distributing of the neutron flux about the time, space and energy for a kind of non-stationary neutron transport problem whose scale of time, space and energy is far less than that of the average transport problem. For such problems, it is quite difficult to reach the precise result with Monte Carlo (MC) non-biased transport calculation, and then how to get the global solution precisely with MC is put forward. However, most of existing MC variance reduction skills are designed for calculating the local quantity of the steady system, which are not adapted to be applied in this problem except the implicit capture. Therefore the method of MC variance reduction for global solution requires to be studied and developed.

*Corresponding author. *Email addresses:* xu_haiyan@iapcm.ac.cn (H. Xu), huang_canada@126.com (Z. Huang), shaohuicai@126.com (S. Cai)

The study on global solution traces back to the 1990s. Prigarin [1], Voytishek [2] and Heinrich [3, 4] obtained some theoretical results. Spanier et al. [5–7], Cooper and Larsen [8], and Shangguan [9] developed several MC methods which have been applied successfully to some simple models.

Zero variance theory points out a direction of designing the MC simulating scheme, which is capable of reducing the variance of the objective if the approximate importance function is to be found. Based on this idea, different kinds of MC variance reduction techniques were presented by many scholars, such as Booth [10–13], Liu and Gardner [14], Tang [15, 16], Turner and Larsen [17], and Wagner and Haghighat [18], but their research fields were only limited in the local solution of the steady system.

For global MC variance reduction, a scheme [19] is proposed based on zero variance theory through breaking up the whole into parts and coupling MC method with discrete ordinates method. Its core is MC transport importance biased calculation guided by the approximate importance function obtained from the deterministic calculation. This guidance behaves mainly at two aspects: to guide the source function sampling by direct biasing and to guide the transform function sampling by indirect biasing. How about the effectiveness of directly biasing the source function and indirectly biasing the transform function? It is necessary to study and analyze its respective effectiveness to better understand the MC transport importance biased calculation.

The remainder of this paper is organized as follows. The scheme of MC transport importance biased calculation and its decomposed strategy are outlined first. Then the choice of representative objectives is discussed. In Section 4, the numerical results are presented and analyzed statistically. Section 5 summarizes the results of this paper with a conclusion.

2 Biased calculation scheme and its decomposed strategy

The integral Boltzmann transport equation for particle emission density in the phase-space $\vec{P}=(\vec{r},E,\vec{\Omega},t)$ is given by

$$Q(\vec{P})=S(\vec{P})+\int Q(\vec{P}')K(\vec{P}'\rightarrow\vec{P})d\vec{P}', \tag{2.1}$$

where $K(\vec{P}'\rightarrow\vec{P})$ is the transform function, $K(\vec{P}'\rightarrow\vec{P})d\vec{P}$ is the expected number of particles emerging in $d\vec{P}$ about $\vec{P}$ from an event in $\vec{P}'$, and $S(\vec{P})$ is the source function or the source density. This is also the starting equation of MC simulation, in which there are two terms contributing to the particle emission density: one is from the source term, and the other from the transform term.

The scheme of MC transport importance biased calculation is developed based on the zero variance theory. Let $\varphi^+(\vec{P})$ be the importance function with respect to the required objective about the emission density. Multiply Eq. (2.1) by $\varphi^+(\vec{P})/\int S(\vec{P})\varphi^+(\vec{P})d\vec{P}$ and