Commun. Comput. Phys.
doi: 10.4208/cicp.070710.100111a

Vol. **10**, No. 5, pp. 1257-1279
November 2011

A High Order Spectral Volume Formulation for Solving Equations Containing Higher Spatial Derivative Terms: Formulation and Analysis for Third Derivative Spatial Terms Using the LDG Discretization Procedure

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Received 7 July 2010; Accepted (in revised version) 10 January 2011

Available online 2 August 2011

Abstract. In this paper, we develop a formulation for solving equations containing higher spatial derivative terms in a spectral volume (SV) context; more specifically the emphasis is on handling equations containing third derivative terms. This formulation is based on the LDG (Local Discontinuous Galerkin) flux discretization method, originally employed for viscous equations containing second derivatives. A linear Fourier analysis was performed to study the dispersion and the dissipation properties of the new formulation. The Fourier analysis was utilized for two purposes: firstly to eliminate all the unstable SV partitions, secondly to obtain the optimal SV partition. Numerical experiments are performed to illustrate the capability of this formulation. Since this formulation is extremely local, it can be easily parallelized and a h-p adaptation is relatively straightforward to implement. In general, the numerical results are very promising and indicate that the approach has a great potential for higher dimension Korteweg-de Vries (KdV) type problems.

AMS subject classifications: 65

Key words: Spectral volume, LDG, higher spatial derivative terms, KDV, Fourier analysis.

1 Introduction

The spectral volume (SV) method was originally developed by Wang, Liu et al. and their collaborators for hyperbolic conservation laws on unstructured grids [20, 29–33]. The spectral volume method is a subset of the Godunov type finite volume method, which has been evolving for decades and has been a starting block for the development of a plethora of methods such as the k-exact finite volume [5, 8], MUSCL (Monotone

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Upstream-centered Schemes for Conservation Laws) [27, 28], and the essentially non-oscillatory (ENO) [1, 11] methods. The spectral volume method can be viewed as an extension of the Godunov method to higher order by adding more degrees-of-freedom (DOFs) in the form of subcells in each cell (simplex). The simplex is referred to as a spectral volume (SV) and the subcells are referred to as control volumes (CV). Every simplex in the SV method consists of a "structured" arrangement of the above mentioned subcells (CVs). As in the finite volume method, the unknowns (or DOFs) are the subcell-averaged solutions. A finite volume procedure is employed to update the DOFs. The spectral volume method shares many similar properties with the discontinuous Galerkin (DG) [6, 7] and the spectral difference (SD) [18, 24] methods, such as discontinuous solution space, sharing multiple degrees of freedom within a single element and compactness. They mainly differ on how the DOFs are chosen and updated. Since the DG is a derivative of the finite element method, most implementations use the elemental nodal values as DOF, though some researchers use the equally valid modal approaches. Although both of the above approaches are mathematically identical, at least for linear equations, different choices of DOFs are used by various researchers result in different efficiency and numerical properties. The spectral volume being a derivative of the finite volume has subcell averages as its DOF while the spectral difference has point wise values as DOF. In terms of complexity, DG requires both volume and surface integrations. In contrast, SV requires only surface integrations and the SD requires differentiations.

The SV method was successfully implemented for 2D Euler [32] and 3D Maxwell equations [20]. The quadrature free formulation was implemented by Harris et al. [9]. A *h*-*p* adaptation was also carried out in 2D [10]. Recently Sun et al. [25] implemented the SV method for the Navier Stokes equations using the LDG [7] approach to discretize the viscous fluxes. Kannan and Wang [14, 17] conducted some Fourier analysis for a variety of viscous flux formulations. Kannan implemented the SV method for the Navier Stokes equations using the LDG2 (which is an improvised variant of the LDG approach) [15] and DDG approaches [16]. Even more recently, Kannan extended the SV method to solve the moment models in semiconductor device simulations [12, 13].

In this paper, we develop a formulation for solving equations containing third spatial derivative terms in a SV context. This formulation borrows ideas from Yan et al. [34, 35] for efficiently implementing the LDG method. A linear Fourier analysis is performed to test the accuracy of this formulation. The Fourier analysis was utilized for two purposes: firstly to eliminate all the unstable SV partitions, secondly to obtain the optimal SV partition. The maximum allowable non-dimensional time step was determined for these optimal partitions.

The paper is organized as follows. In the next section, we review the basics of the SV method. The LDG formulation for high order spatial derivatives is presented in Section 3. A detailed linear analysis is performed for the LDG formulation in Section 4. Section 5 presents with the different test cases conducted in this study. Finally conclusions from this study are summarized in Section 6.