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Stability of Finite Difference Discretizations of Multi-Physics Interface Conditions

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Abstract. We consider multi-physics computations where the Navier-Stokes equations of compressible fluid flow on some parts of the computational domain are coupled to the equations of elasticity on other parts of the computational domain. The different subdomains are separated by well-defined interfaces. We consider time accurate computations resolving all time scales. For such computations, explicit time stepping is very efficient. We address the issue of discrete interface conditions between the two domains of different physics that do not lead to instability, or to a significant reduction of the stable time step size. Finding such interface conditions is non-trivial.
We discretize the problem with high order centered difference approximations with summation by parts boundary closure. We derive L^2 stable interface conditions for the linearized one dimensional discretized problem. Furthermore, we generalize the interface conditions to the full non-linear equations and numerically demonstrate their stable and accurate performance on a simple model problem. The energy stable interface conditions derived here through symmetrization of the equations contain the interface conditions derived through normal mode analysis by Banks and Sjögreen in [8] as a special case.

AMS subject classifications: 65M06, 65M12, 74F10

Key words: Fluid-structure interaction, finite difference method, summation by parts, multi-physics interface condition.

1 Introduction

This work will consider numerical simulation of multi-physics systems where two, or more, physics models are solved on different parts of a computational domain. These different multi-physics domains are assumed to be separated by well-defined interfaces. Coupling conditions which join the various sub-domains are defined on these interfaces.

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In the literature, there is a large body of work relating to numerical treatment of this type of fluid-structure interface. By far the most common approach is to apply material motions from the solid domain as boundary conditions to the fluid while using the fluid stresses as boundary conditions on the solid. However, this approach can become problematic from a stability perspective for certain cases. As a result, implicit, sometimes referred to as monolithic, approaches have often been adopted. Such schemes are effective, but can introduce additional difficulties in terms of linear/nonlinear solvers and pre-conditioners.

Recent work in [2,7,8] has shown that more symmetric approaches to interface condition imposition can result in favorable approximations, possibly with stability across all ranges of material parameters. The main purpose of the current work is to discuss the well-posedness of the continuous linearized fluid-structure problem and introduce a summation-by-parts discretization which mimics the energy behavior of the continuous operators. The operators which are thus derived have similar structure to those found in [8]. We will verify the accuracy of this new approach via manufactured solutions and apply the schemes to a nontrivial problem of a Navier-Stokes fluid with an elastic-plastic solid.

The example studied here will be fluid/structure interaction in one space dimension, but the ideas are intended as more generally applicable. However, the techniques employed in this work have an impact on the discretization choices available for extension to, for example, two space dimensions. This is an important point and so we provide a brief discussion of these issues. The eventual numerical discretization of the governing equations and interface conditions investigated here requires a set of interface aligned grids. That is to say that the interface defining the boundary between two, or more, physics sub-domains must be represented in both computational sub-domains. This is shown graphically in Fig. 1. Here, the fluid equations are discretized on the blue grids and the solid is discretized on the red grid. The requirement that both computational sub-domains align with the material interface implies that other techniques are required to deal with external boundaries. In Fig. 1, we indicate that an overset grid approach [5] is used to treat the fluid domain, while a structured deforming grid is used for the solid. Other options are of course possible and include embedded boundaries, overset grids, unstructured grids, or others. The key requirement as it relates to this work is that the fluid and solid are discretized on grids which align to the material interface.

The remainder of this work is structured as follows. Section 2 describes the equations in full generality. In Section 3, we perform an energy estimate of the one dimensional continuous problem and prove that the standard interface conditions lead to a well-posed problem with decreasing energy. Section 4 performs the same analysis for finite difference discretizations that satisfy the summation by parts principle. We find energy stable discretizations and give explicit formulas for the discrete interface conditions. Section 5 shows numerical examples in one space dimension. The method of manufactured solution is first used to verify the implementation and to study the numerical convergence order obtained for finite difference schemes of different formal accuracies. Finally, we