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An Optimized Correction Procedure via Reconstruction Formulation for Broadband Wave Computation

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Abstract. Recently, a new differential discontinuous formulation for conservation laws named the Correction Procedure via Reconstruction (CPR) is developed, which is inspired by several other discontinuous methods such as the discontinuous Galerkin (DG), the spectral volume (SV)/spectral difference (SD) methods. All of them can be unified under the CPR formulation, which is relatively simple to implement due to its finite-difference-like framework. In this paper, a different discontinuous solution space including both polynomial and Fourier basis functions on each element is employed to compute broad-band waves. Free-parameters introduced in the Fourier bases are optimized to minimize both dispersion and dissipation errors through a wave propagation analysis. The optimization procedure is verified with a mesh resolution analysis. Numerical results are presented to demonstrate the performance of the optimized CPR formulation.

AMS subject classifications: 76

Key words: CPR (correction procedure via reconstruction), hybrid discontinuous space, wave propagation analysis, unstructured meshes.

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1 Introduction

In the last two decades, there has been a surge of research activities on high-order methods capable of solving the Navier-Stokes equations on unstructured grids. For a review of some of these activities, the readers can refer to [11, 44]. Many powerful high-order numerical algorithms have been developed, e.g. the spectral element method [30], multi-domain spectral method [20, 21], k-exact finite volume method [4], WENO methods [14], discontinuous Galerkin (DG) method [5, 8, 9], high-order residual distribution

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methods [1], spectral volume (SV) [27, 33, 43, 48, 49] and spectral difference (SD) methods [16, 26, 29, 34, 35, 50, 51]. A new discontinuous formulation named Correction Procedure via Reconstruction (CPR) was recently developed in [17], and extended to simplex meshes in [46]. The degrees-of-freedom (DOFs) are the state variables at solution points (SPs) in the CPR formulation, where the differential form of the governing equation is solved. As a result, explicit surface and volume integrals are avoided. The CPR formulation is among the most efficient discontinuous methods in terms of the number of operations.

The stability and accuracy of the discontinuous high-order methods depend on how the solutions are approximated and the weighting functions are chosen. The piecewise polynomial space is commonly chosen for convection problems. However, piecewise polynomials may not provide the best approximation for some PDEs and initial/boundary conditions. We now list some examples in the literature. The locally divergence-free polynomial space was used in the DG method to solve the Maxwell equations and better results were achieved compared to the classical piecewise polynomial space in [7, 22–24]. Exponential functions were proposed to solve singular perturbation problems by Kadalbajoo and Patidar [19] and by Reddy and Chakravarthy [31]. Non-polynomial spaces were used in the local essentially non-oscillatory (ENO) reconstruction for solving hyperbolic conservation laws in [6]. Another work is the use of exponential functions near a boundary, and the use of trigonometric functions for highly oscillatory problems, as shown by Yuan and Shu [52].

In the present study, a hybrid space including both polynomial and Fourier functions are employed to resolve broadband wave propagation problems. Fourier functions are used such that the method can exactly represent waves at certain wave numbers, while polynomial functions are employed to preserve a certain order of accuracy. Free-parameters introduced in the Fourier functions are optimized by mimicking the dispersion-relation-preserving (DRP) method to minimize both dispersion and dissipation errors [25, 32, 36, 37, 54, 55]. The basic idea of the DRP method is to optimize the scheme coefficients for the high resolution of short waves with respect to the computation grid instead of the truncation errors. The present method is named a frequency optimized CPR formulation (FOCPR) in the present paper.

Fourier analyses have been preformed to investigate the dispersive and dissipative errors for finite difference and finite volume methods [25,32,36,37,54,55]. Hu [15] applied it for the DG method, and Van den Abeele et al. [42] carried out such an analysis for the 1D spectral volume method. In this paper, the accuracy and stability properties of the CPR method with the hybrid spaces are assessed by following similar techniques. A mesh resolution analysis is also performed to study the points-per-wavelength (PPW) requirement to achieve a certain accuracy following the procedure in [18, 53] in order to verify the optimization procedure. Several numerical tests are performed, which show that the FOCPR method can resolve broadband waves more accurately than the original CPR method.

This paper is organized as follows. For the sake of completeness, the framework