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Forward Scattering and Volterra Renormalization for Acoustic Wavefield Propagation in Vertically Varying Media

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Abstract. We extend the full wavefield modeling with forward scattering theory and Volterra Renormalization to a vertically varying two-parameter (velocity and density) acoustic medium. The forward scattering series, derived by applying Born-Neumann iterative procedure to the Lippmann-Schwinger equation (LSE), is a well known tool for modeling and imaging. However, it has limited convergence properties depending on the strength of contrast between the actual and reference medium or the angle of incidence of a plane wave component. Here, we introduce the Volterra renormalization technique to the LSE. The renormalized LSE and related Neumann series are absolutely convergent for any strength of perturbation and any incidence angle. The renormalized LSE can further be separated into two sub-Volterra type integral equations, which are then solved noniteratively. We apply the approach to velocity-only, density-only, and both velocity and density perturbations. We demonstrate that this Volterra Renormalization modeling is a promising and efficient method. In addition, it can also provide insight for developing a scattering theory-based direct inversion method.

AMS subject classifications: 35R30, 45D05, 45G10, 45Q05, 65R20, 81Q15, 81U40, 86A15

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1 Introduction

Scattering theory is a powerful method for analyzing wave propagation in a given medium [1, 30]. It relates the propagation of the wave in a general medium with the propagation of the wave in a reference medium and a perturbation operator which characterizes the difference between the actual and reference media. It is well known that the relation between the actual wave-field and the perturbation is nonlinear and can be represented as an infinite series (scattering series or Born-Neumann series). The main use of the forward scattering series arose from the application of scattering theory to solving inverse problems. Based on the early work of Jost and Kohn [6], Moses [14], Razavy [20] and Prosser [17], Weglein and co-workers developed the inverse scattering series (ISS) method [22,24–26,28,37]. The ISS has proven to be a good framework for a wide range of seismic problems, e.g., multiple attenuation, imaging, direct nonlinear inversion. However, the forward scattering using Born-Neumann series method suffers from the issue of convergence [18]. Convergence properties of the forward scattering series for different types of acoustic medium have been extensively studied. Maston shows that the convergence occurs only for a ratio less than $\sqrt{2}$ between the reference and the actual velocities for a 1-D acoustic medium. Later Maston [12] and Nita [27] extended the study to a two dimensional vertically varying acoustic medium. They demonstrated that the forward scattering series only converges for either limited velocity contrast or limited incident angle. The same limited convergence condition was shown by Ramirez and Otens for the acoustic multi-parameter case [19]. To overcome the limitation of convergence, various methods have been proposed. Nita [16] showed that, using a Padé approximation, it is possible to extend the convergence properties of the forward scattering series to any velocity contrast in acoustic media. In addition, there also exist several partial summation techniques where only those nonlinear components that contribute to specific events (e.g., primary, multiple) are retained. One is the De Wolf approximation (DWA) [2, 3], introduced to seismic problems by Wu and co-workers [31–34]. The DWA is a multiple forward scattering and single backscattering approximation after reordering and renormalization of the Born series. Another nonlinear Born series partial summation is that derived by Weglein and co-workers [22, 24, 28], who divided the full series into various subseries, where each subseries is responsible for a single task [28]. Based on that idea, Innanen also derived a nonlinear integral transform relationship between the primary data and the earth model to model primary events [4, 5].

However, the convergence issue of the forward scattering problem can be overcome by employing the Volterra renormalization technique to the LSE. The Volterra renormalization technique was first proposed by Kouri and co-workers for studying quantum scattering [21]. Later, it was applied to a 1-D acoustic medium for modeling wave propagation with normal plane wave incident [8, 9]. In this paper, we extend this method to a layered medium, for which both velocity and density are allowed to vary. With the renormalization, the Fredholm type of the LSE is transformed to a Volterra type. It has been proved that solutions related to the renormalized LSE converge for any perturbation for