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Dispersion Error Analysis of Stable Node-Based Finite Element Method for the Helmholtz Equation

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Abstract. Numerical dispersion error is inevitable when the finite element method is employed to simulate acoustic problems. Studies have shown that the dispersion error is essentially rooted at the "overly-stiff" property of the standard FEM model. To reduce the dispersion error effectively, a discrete model that provides a proper softening effects is needed. Thus, the stable node-based smoothed finite element method (SNS-FEM) which contains a stable item is presented. In this paper, the SNS-FEM is investigated in details with respect to the pollution effect. Different kinds of meshes are employed to analyze the relationship between the dispersion error and the parameter involved in the stable item. To ensure the SNS-FEM can be applied in the practical engineering problems effectively, the relationship is finally constructed based on the hexagonal patch for the commonly used unstructured mesh is very similar to it. By minimizing the discretization error, an optimal parameter equipped in this novel SNS-FEM is formulated. Both theoretical analysis and numerical examples demonstrate that the SNS-FEM with the optimal parameter reduces the dispersion error significantly compared with the FEM and the well performed GLS, especially for mid- and high-wave number problems.

AMS subject classifications: 35J05, 65C20, 65N30, 65N22, 68U20, 81U30
Key words: Acoustic, numerical method, the stable node-based smoothed finite element method (SNS-FEM), dispersion error, hexagonal patches.

1 Introduction

Nowadays, with the increasing demands on the acoustic performance of enclosed cavities, such as the automotive passenger compartments and aircraft cabins, careful con-

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siderations must be given in designing these sophisticated products. As the analytical solutions are unavailable for problems with complex configurations and various boundary conditions, researchers often resort to the numerical methods when performing the acoustic analysis for these cases. Currently, the standard finite element method (FEM) [1–4] and boundary element method (BEM) [5, 6] are the two widely-used and well-developed numerical tools in simulating acoustic problems. For exterior acoustic problems, the BEM shows great advantages since it allows the simulation of fields in unbounded domain. However, when it comes to interior acoustic problems, researchers often chose the FEM for its simplicity and efficiency.

Numerical methods, when applied to time-harmonic wave propagation problems, will unavoidably cause the dispersion error [7–11]. At higher frequencies, the dispersion tends to accumulate, resulting in completely erroneous results [12, 13]. Finite element users often believe the rule of the thumb, which prescribes that 7 or 10 points per wavelength are needed to obtain a reasonable result when linear element is employed [14]. However, this criterion is not always true. In most cases, the error of numerical solutions often grows with the increase of wave number even the criterion is satisfied. Although the use of refined meshes could alleviate the dispersion effect, a large amount of computational cost has to be consumed, which produces great burdens for large scale three-dimensional practical engineering problems.

In order to reduce the dispersion in the high frequency range, various numerical improvements have been proposed in the last decades. The first approach is the stabilized finite element method [2, 3] which was developed by the modifications of the classical Galerkin method, such as the Galerkin/least-squares finite element method (GLS) [14–16], the residual-free bubbles method (RFB) [17] and the quasi-stabilized finite element method (QSFEM) [18]. Another effective strategy is the high-order finite element method, including the p-version FEM [19–22] and the partition of unity method (PUM) [23]. Although these methodologies are very effective in controlling the pollution effect, the dispersion still exists when resorting to these methods for acoustic analysis. With the development of mesh-free methods, the element-free Galerkin method (EFGM) [24, 25], the multiresolution reproducing kernel particle method (RKPM) [26] and the radial point interpolation method (RPIM) [27, 28] have already been used to analyze the acoustic problems by researchers in recent years. Compared to the classical FEM, these different mesh-free strategies improve the computational accuracy significantly, however, the dispersion still cannot properly removed in the general two- and three-dimensional acoustic problems.

The dispersion error increases conspicuously with the increasing wave number due to the overly-stiff property of the FEM model [29–33]. In order to eliminate the dispersion error effectively, a discretized model with proper stiffness is needed [29–32]. For this purpose, a strain smoothing method was proposed by Chen [34, 35], and a generalized gradient smoothing (GGS) technique [36–38] has been further formulated by Liu. The GGS has been extensively used in analysing solid mechanics problems and show significant superiority since it can offer a sufficient softening effect. Based on the node-based