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Stochastic Multiscale Heat Transfer Analysis of Heterogeneous Materials with Multiple Random Configurations

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Abstract. This study presents a new stochastic multiscale analysis approach to analyze the heat transfer performance of heterogeneous materials with random structures at different length scales. The heterogeneities of the materials are taken into account by periodic layouts of unit cells, consisting of randomly distributed inclusion dispersions and homogeneous matrix on the microscale and mesoscale. Based on the reiterated homogenization, a novel unified micro-meso-macro stochastic multiscale formulation is established and the scale gap is correlated by means of two-scale asymptotic expansions. Also, the stochastic multiscale formulae for computing the effective thermal property and temperature field are derived successively. Then, the stochastic prediction algorithm coupled with the finite element method is brought forward in details. The accuracy of the implemented stochastic multiscale analysis is verified by comparing the results against the experimental data for three scales heterogeneous materials with several different material combinations. The comparison demonstrates the usability of the proposed stochastic multiscale method for the determination of the thermal behaviors. This study offers a unified multiscale framework that enables heat transfer behavior analysis of heterogeneous materials with multiple random configurations.

AMS subject classifications: 35B27, 34E13, 74Q05, 80A20

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1 Introduction

Many natural and synthetic materials exhibit a random structure at more than two length scales, such as the fiber reinforced concrete, which is a typical three-scales random composite with fine grained concrete as the matrix and short fibers as the reinforcement [1]. The advantageous properties of these composites are largely due to the hierarchical and functional relationships between each pair of adjacent two scales. Since heterogeneous materials with multiple random configurations are ubiquitous in engineering applications, it is important to understand the thermal properties and responses, which can arise at a myriad of length scales [2]. Thus, the multiple scales coupled analysis is necessary to adequately study the heat transfer performances of heterogeneous materials with random structures at different length scales.

With the rapid development of computer science and technology, numerical modeling becomes a useful tool for predicting the effective behavior of heterogeneous materials [3–5]. It is difficult to perform a direct numerical simulation of large-scale structures by means of micro-analysis, since an extremely fine spatial discretization mesh is required to capture the effects of the microscale heterogeneities, which may exceed the capacity of modern computers [6]. Because of the great difference in scale of macroscopic structures and inhomogeneous microstructures of materials, reiterated homogenization techniques [7] for treatment of up-scaling procedure on different length scales are becoming indispensable for understanding the influence of microscale heterogeneities on macroscopic behaviors of heterogeneous materials. The theory behind this method is described and developed, for example, in the papers of Allaire and Briane [8], Meunier and Van Schaftingen [9], Trucu et al. [10], Zhao [11] and Pastukhova and Tikhomirov [12]. There also exist numerous articles, which have applied the reiterated homogenization method to determine the effective mechanical and thermal properties of heterogeneous materials. Telega et al. [13] obtained the effective macroscopic elastic moduli of a compact bone by using reiterated homogenization. Almqvist et al. [14] devoted to studying the combined effect that arises due to surface texture and surface roughness in hydrodynamic lubrication by using the theory of reiterated homogenization with three scales. Dimitrienko et al. [15] developed a determination method based on reiterated homogenization for effective elastic characteristics of composites with a multiscale hierarchical structure. Rohan et al. [16] studied the two-level homogenization of Stokes flows in the microporous material which is drained into the mesoscopic pores. Ramírez-Torres et al. [17] proposed a multiple scales asymptotic homogenization approach to study the effective properties of hierarchical composites with periodic structure at different length scales. Engquist and Ying [18] considered the numerical evaluation of effective coefficients for multiscale homogenization problems and proposes a highly efficient algorithm for a certain class of reiterated homogenization problems of practical importance. After that, Rodríguez et al. [19] obtained the effective thermal conductivity parameters of heat transfer problem in heterogeneous media with multiple spatial scales and perfect thermal contact between different phases by applying the reiterated homogenization method. And the ef-