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Stability Analysis for Wave Simulation in 3D Poroelastic Media with the Staggered-Grid Method

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Abstract. In this paper, a new approach is proposed to analyse the stability of high-order staggered-grid finite difference schemes for the three-dimensional (3D) poroelastic wave propagation. The standard staggered-grid schemes with different order accuracy in space are constructed based on the first-order hyperbolic velocity-stress system of the governing equations (i.e., Biot's equations). The new analysis method is based on von Neumann analysis. The obtained 3D stability is an explicit restriction for time step, which only depends on the coefficients of the difference operators and the material parameters of poroelastic media and so it can be computed easily. Moreover, the analysis has good generality and can be applied directly to the staggered-grid schemes for 3D elastic wave. Numerical computations with the perfectly matched layer in split formation are implemented to illustrate the effectiveness of the schemes for 3D poroelastic wave propagation. The method in this paper can be expected to analyse the stability for other staggered-grid schemes.

AMS subject classifications: 35L05, 65C20, 65N12, 65N06, 74S20

Key words: 3D, stability analysis, poroelastic media, wave propagation, staggered-grid.

1 Introduction

Wave propagation has important applications in geophysics. For instance, the detection of underground structures is based on the acoustic and elastic wave propagation [1]. In order to know the phenomenon of wave propagation well, one needs to numerically solve the wave equations. There are many existing numerical methods to solve the wave equations and here we only list a few references. They include the finite difference (FD)

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method [15, 39, 46], the pseudo-spectral method [26, 32, 55], the finite element method [2, 16, 42], the spectral element method [17, 33], the discontinuous Galerkin (DG) method [13, 31] and the finite volume method [58, 59]. Each method has its own advantages and disadvantages. Among of these methods, the FD method is still a popular and widely used method because of its high computational efficiency and relatively easiness of code implementation.

Wave propagation based on acoustic or elastic model has been proved a great success in oil geophysical exploration. In such models, properties of a pore fluid such as density, bulk modulus, saturation and viscosity have been ignored. Despite the acoustic or elastic model, a porous medium is a more realistic model. By using a model based on the poroelastic wave equations, the effects of fluid, pressure, porosity and permeability between phases can be taken into account and provide more accurate solutions that can not be obtained through the use of pure elastic or acoustic model. Numerical simulation of poroelastic wave propagation has received more attention for decades. A review can be found in [11]. In [40], the spectral element was applied to model waves in poroelastic media. In [21, 23], the DG method was applied to simulate poroelastic waves. In [52], a symplectic method was proposed for solving elastic wave equations in 2D porous media. The staggered-grid (SG) FD scheme is a popular numerical method for solving partial differential equations. This method originally developed by [49, 50] for elastic wave allows different variables and constants in the equations to be discretized at different locations on the grid. It has been applied to solve poroelastic wave equations widely and successfully, for example, see the references [9, 19, 37, 38, 45, 51, 53]. Recently, the 3D rotated SG scheme and the optimal SG implicit scheme are designed to model wave propagation in poroelastic media [30, 44].

The stability of staggered-grid FD schemes for 3D poroelastic wave equations is still a worth studying problem. In [37], the stability in one dimensional case is derived formally. In [30], the 2D stability for the optimal implicit FD scheme is investigated. However, the stability in 3D case has not been formally given. From numerical experiments in both two and three dimensions and from analogy with elastic case, [37] suggested the stability in higher dimensions can be obtained approximately using the 1D stability divided by a factor $\sqrt{d}$, here d is the number in dimension. This empirical criterion is applied by [44], which the 3D stability is yielded based on the 1D stability criterion multiplying by $1/\sqrt{3}$. In [44], the 2D explicit stability is obtained by using the dispersion analysis method. However, to obtain the explicit 3D stability by using the dispersion analysis method is difficult because the related constants in stability "contain up to several hundred terms and are not easily determined" [44]. In this paper, we develop a new effective approach for analysing the stability of high-order SG schemes for 3D poroelastic wave equations. The obtained stability is an explicit restriction for time step, which only depends on the material parameters and the coefficients in the schemes. So it can be computed easily. Moreover, the developed approach has good generality and the simplification to both 2D and 3D elastic wave equations is verified. In this paper, we focus on the 3D stability analysis and the dispersion analysis can be found in [44]. It is not the main concern of