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A Simple Strategy for Multi-Material Diffusion and Its Application to Three-Temperature Multi-Material Flows

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Abstract. This work is devoted to the numerical approximation of three-temperature multi-material hydrodynamics. Such systems are subject to stiff phenomena which require specific care during the discretization. In particular, the so-called discrete equation method is here applied to the radiation transport, in the optically-thick limit. This strategy is shown to be accurate in the presence of in-cell interfaces while being simpler than standard interface reconstruction techniques. It is then incorporated into a three-temperature multi-material scheme whose implicit temporal discretization is based on convex combinations. Stiff test cases eventually establish the scheme's robustness.

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1 Introduction

1.1 Three temperature multi-material hydrodynamics

In applications such as Inertial Confinement Fusion [6] or astrophysics, the materials under scrutiny are usually plasmas. The behavior of strongly ionized plasmas is characterized by different thermalization time scales. Ions and electrons individually reach thermal equilibrium on characteristic times much shorter than those necessary for the mixture thermalization. If the hydrodynamic time scale falls between the two, a two-temperature description of the plasma is necessary; ions and electrons each have their own temperature but both temperatures need not be equal. Likewise, in the presence of

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*Corresponding author. *Email address:* `sebastien.guisset@cea.fr` (S. Guisset) a strong radiation field, photons need their own description which is here given by the grey diffusion approximation. Such an approximation is relevant in the optically thick limit if radiation thermalization is assumed [35]. The resulting three different temperatures are coupled through collisions between ions and electrons and between electrons and photons. These couplings can be arbitrarily stiff and the single-temperature Euler equations are recovered in the case of instantaneous relaxation. Besides, radiation transport accounts for the propagation of photons throughout the domain and can also display a stiff behavior. Diffusion on the ionic and electronic energies exists as well but are here neglected.

The physical description can be supplemented with an additional multi-material layer. Multi-material aspects are essential to properly describe complex mixtures with contrasted physical characteristics (e.g. mixtures of gases and solids). The most common approach to derive multi-material equations is that of conditional averaging procedures [18,25,42]. It produces averaged quantities describing the behavior of the flow, as well as additional correlation terms which encapsulate the remaining details. These correlation terms are often neglected as a first approximation as they are deemed minor, at least outside of shocks. If this approach has proven its efficiency in describing single-temperature multi-material flows, its application to radiation transport is not trivial. Indeed, averaged fluxes only express radiation transport separately within each material while the associated correlation term deals with the coupling between different materials. While radiation transport in between materials is essential for weakly opaque mixtures of materials, its expression remains unclear and heavily mixture-dependant. Alternatively, if it is neglected as a first approximation, then different materials are no longer coupled through temperatures, thus leading to questionable results.

1.2 Numerical strategy

1.2.1 Multi-material radiation transport or diffusion

The discretization of the multi-material radiation transport consists in one of the main contribution of this work. As explained above, its expression at the continuous level is a difficult task. Still, its discretization is possible and different strategies exist in the literature. Interface reconstruction techniques [14, 21, 37] estimate the interface between materials inside mixed cells. Mixed cells are then separated into pure cells and the diffusion operator is discretized with any single material strategy. At their best, these techniques allow for a very accurate description of the mixture. However, complex topology of the flow requires ad hoc strategies [22] and interface reconstruction fails at describing dispersed phases. The treatment of three or more materials is usually not invariant under permutation of materials and depends on an arbitrary order [28]. Finally, the computational cost, especially in three dimensions, can be significant. Alternatively, homogenization methods [13] assume that mixture of different materials act as a single equivalent material. Such methods are inexpensive but are usually not considered reliable [13, 27]. Apart from obvious accuracy issues on the fluxes, homogenization methods are built on