COMMUNICATIONS IN COMPUTATIONAL PHYSICS
Vol. 6, No. 2, pp. 319-341

Commun. Comput. Phys.
August 2009

A Multilevel Domain Decomposition Approach for Studying Coupled Flow Applications

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Received 25 October 2007; Accepted (in revised version) 25 July 2008

Available online 15 December 2008

Abstract. In this paper, a multilevel domain decomposition approach based on multi-grid methods for obtaining fast solutions for coupled engineering flow applications arising on complex domains is presented. The proposed technique not only allows solutions to be computed efficiently at the element level but also helps us to achieve proper accuracy, load balancing and computational efficiency. Numerical results presented demonstrate the robustness of the proposed technique.

AMS subject classifications: 65N30, 65N55, 65N12

Key words: Finite element methods, multigrid methods, domain decomposition.

1 Introduction

Over the last decade, there have been significant advances in developing solution methodologies for studying complex dynamics of coupled processes arising in a variety of applications that involve multiple interactions between flow, temperature and structures [3, 4, 9–11, 25, 33, 34, 37]. Domain decomposition techniques with non-matching grids have become increasingly popular in studying such coupled processes [2, 5, 29, 30]. In particular, they help achieve fast and accurate solutions to various applications involving coupled processes when used in conjunction with multigrid techniques [19,32]. They also allow coupling of different subdomains with nonmatching grids and different discretization techniques and the solution can be efficiently implemented even over parallel architectures.

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The purpose of this paper is to introduce a flexible domain decomposition approach that involves multigrid algorithm that will be used to study different engineering applications that involve flow mechanics. The first two applications involve flows through a channel with square cavities. The third and the fourth applications involve interaction between flow and large deforming structures.

2 Model and governing equations

We denote by $H^s(O)$, $s\in\Re$, the standard Sobolev space of order s with respect to the set O, which is either the flow domain Ω, or its boundary Γ, or part of its boundary. Hence, we associate with $H^m(O)$, its natural norm $\|\cdot\|_{m,O}$. For $1\le p<\infty$, the Sobolev space $W^{m,p}(O)$ is defined as the closure of $C^\infty(O)$ in the norm

$$\|f\|^p_{W^{m,p}(O)}=\sum_{|\alpha|\le m}\int_O\left|\left(\frac{\partial}{\partial x}\right)^\alpha f(x)\right|^p dx.$$

The closure of $C_0^\infty(O)$ under the norm $\|\cdot\|_{W^{m,p}(O)}$ will be denoted by $W_0^{m,p}(O)$. Whenever possible, we will neglect the domain label in the norm.

For vector-valued functions and spaces, we use boldface notation. For example, $\mathbf{H}^s(\Omega)=[H^s(\Omega)]^n$ denotes the space of $\Re^n$-valued functions such that each component belongs to $H^s(\Omega)$. Also we denote the space of square integrable functions having zero mean over Ω by $L_0^2(\Omega)$ and the space of solenoidal functions

$$\mathbf{V}(\Omega)=\{\boldsymbol{u}\in\mathbf{H}^1(\Omega)\,|\,\nabla\cdot\boldsymbol{u}=0\}.$$

For $\Gamma_1\subset\Gamma$ with non-zero measure, we also consider the subspace

$$\mathbf{H}^1_{\Gamma_1}(\Omega)=\{\boldsymbol{v}\in\mathbf{H}^1(\Omega)\,|\,\boldsymbol{v}=\vec{0}\quad\text{on }\Gamma_1\}.$$

Also, we denote $\mathbf{H}^1_0(\Omega)=\mathbf{H}^1_\Gamma(\Omega)$. For any $\boldsymbol{v}\in\mathbf{H}^1(\Omega)$, we write $\|\nabla\boldsymbol{v}\|$ for the semi-norm. Let $(\mathbf{H}^1_{\Gamma_1})^*$ denote the dual space of $\mathbf{H}^1_{\Gamma_1}$. Note that $(\mathbf{H}^1_{\Gamma_1})^*$ is a subspace of $\mathbf{H}^{-1}(\Omega)$, where the latter is the dual space of $\mathbf{H}^1_0(\Omega)$. The duality pairing between $\mathbf{H}^{-1}(\Omega)$ and $\mathbf{H}^1_0(\Omega)$ is denoted by $<\cdot,\cdot>$.

Let $\boldsymbol{g}$ be an element of $\mathbf{H}^{1/2}(\Gamma)$. It is well known that $\mathbf{H}^{1/2}(\Gamma)$ is a Hilbert space with norm

$$\|\boldsymbol{g}\|_{\frac{1}{2},\Gamma}=\inf_{\boldsymbol{v}\in\mathbf{H}^1(\Omega);\gamma_\Gamma\boldsymbol{v}=\boldsymbol{g}}\|\boldsymbol{v}\|_1,$$

where γ_Γ denotes the trace mapping $\gamma_\Gamma:\mathbf{H}^1(\Omega)\to\mathbf{H}^{1/2}(\Gamma)$. We let $(\mathbf{H}^{1/2}(\Gamma))^*$ denote the dual space of $\mathbf{H}^{1/2}(\Gamma)$ and $<\cdot,\cdot>_\Gamma$ denote the duality pairing between $(\mathbf{H}^{1/2}(\Gamma))^*$ and $\mathbf{H}^{1/2}(\Gamma)$.