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A Unified Gas-Kinetic Scheme for Continuum and Rarefied Flows II: Multi-Dimensional Cases

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Abstract. With discretized particle velocity space, a multi-scale unified gas-kinetic scheme for entire Knudsen number flows has been constructed based on the kinetic model in one-dimensional case [J. Comput. Phys., vol. 229 (2010), pp. 7747-7764]. For the kinetic equation, to extend a one-dimensional scheme to multidimensional flow is not so straightforward. The major factor is that addition of one dimension in physical space causes the distribution function to become two-dimensional, rather than axially symmetric, in velocity space. In this paper, a unified gas-kinetic scheme based on the Shakhov model in two-dimensional space will be presented. Instead of particle-based modeling for the rarefied flow, such as the direct simulation Monte Carlo (DSMC) method, the philosophical principal underlying the current study is a partial-differential-equation (PDE)-based modeling. Since the valid scale of the kinetic equation and the scale of mesh size and time step may be significantly different, the gas evolution in a discretized space is modeled with the help of kinetic equation, instead of directly solving the partial differential equation. Due to the use of both hydrodynamic and kinetic scales flow physics in a gas evolution model at the cell interface, the unified scheme can basically present accurate solution in all flow regimes from the free molecule to the Navier-Stokes solutions. In comparison with the DSMC and Navier-Stokes flow solvers, the current method is much more efficient than DSMC in low speed transition and continuum flow regimes, and it has better capability than NS solver in capturing of non-equilibrium flow physics in the transition and rarefied flow regimes. As a result, the current method can be useful in the flow simulation where both continuum and rarefied flow physics needs to be resolved in a single computation. This paper will extensively evaluate the performance of the unified scheme from free molecule to continuum NS solutions, and from low speed micro-flow to high speed non-equilibrium aerodynamics. The test cases clearly demonstrate that the unified scheme is a reliable method for the rarefied flow computations, and the scheme provides an important tool in the study of non-equilibrium flow.

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1 Introduction

The development of accurate numerical methods for all flow regimes is challenging. It is an important area which is related to the space exploration, vacuum technology, laser development, and many other scientific research and engineering applications. To the current stage, the Direct Simulation Monte Carlo (DSMC) method is the most effective and dominant numerical method for molecular simulation of dilute gases. The main reason for its success is due to its statistical modeling which is consistent with the Boltzmann equation. Based on the Boltzmann equation,

$$f_t+\mathbf{u}\cdot\nabla f=J(f), \tag{1.1}$$

the main feature of the DSMC method is to split the above equation into two processes:

1. relaxation in accordance to the collisional operator of the Boltzmann equation

$$\frac{\partial f}{\partial t}=J(f), \tag{1.2}$$

2. free-molecular transport

$$\frac{\partial f}{\partial t}=-\mathbf{u}\cdot\nabla f. \tag{1.3}$$

A valid physical process which is consistent with the above numerical splitting treatment is that the cell size and time step used in DSMC have to be less than particle mean free path and collision time. Under this condition, the pair of particles chosen for collision in corresponding with the physical frequency of molecular collision, is independent of the distance between particles within the same computational cell. This requirement constraints the extension of the DSMC method to the continuum flow regime, where the cell size used may be many orders larger than the local particle mean free path. Most current research related to the further development of the DSMC method is on the modeling of collision procedure for complicated gas viscosity laws and the reduction of statistical noise due to limited number of particles. On the other hand, due to the particle nature and direct statistical modeling in the DSMC method, the lack of a direction connection with the kinetic equation may evoke certain mistrust of its solution and may lead to certain difficulties in systematic approach to the increase of method's effectiveness. The necessity to construct a close connection between DSMC solution and the solution of kinetic equation is inevitable due to a number of reasons [2]. Numerous solutions have been obtained by DSMC method, but most of them were not repeated with the help of