Commun. Math. Anal. Appl.
doi: 10.4208/cmaa.v4n4.preface

Vol. **4**, No. 4, pp. i-ii
November 2025

Preface

Special Issue on Recent Progresses in Stochastic Differential Equations and Stochastic Partial Differential Equations

Stochastic differential equations (SDEs) has been very well developed since the seminal work of the great Japanese mathematician Kiyosi Itô in the mid 1940s. Since then, SDEs have profound impacts on differential geometry and partial differential equations (PDEs). Facilitated in particular with the theory of PDEs in 1970s, it emerged the fascinating topic of stochastic partial differential equations (SPDEs), mainly due to Etienne Pardoux and John B. Walsh in 1980s. SPDEs are essential for modelling systems where randomness is not just global but varies spatially and lies in their fundamental ability to model systems that evolve unpredictably over time and space. SDEs and SPDEs are not just straightforward extensions of their deterministic counterparts (ODEs and PDEs), they represent a paradigm shift in modelling the natural, engineering, and financial worlds, which are inherently noisy and uncertain. SDEs and SPDEs have incorporated "uncertainty and randomness" directly into the model, moving from idealistic deterministic models to realistic ones that reflect the messy, unpredictable nature of real-world systems, allowing to "quantify uncertainty" and instead of a single predicted path, they provide a "distribution of possible future states" (for instance, a probability distribution for a stock price or a species' population). This is invaluable for risk assessment. They have driven the development of profound new areas of mathematics, including Itô calculus, Malliavin calculus, and the theory of rough paths. Solving them requires a completely different approach than deterministic equations. In conclusion, SDEs and SPDEs are not just mathematical curiosities; they are essential tools for a quantitative understanding of any complex system influenced by randomness over time and space. Their development is a cornerstone of modern applied mathematics.

The main objective of this special issue is to serve as a forum for promoting applicable aspect of SDEs and SPDEs. Wang studied the convergence rate in Wasserstein distance for empirical measure of Markov processes. For diffusion processes on compact manifolds, with reflecting or killing boundary conditions,

sharp convergence rate and renormalization limits been established, characterized by the dimension of the manifold and spectrum of the generator. Xu concerning SDEs forced by stable-like noises, the classical Norris lemma were placed by established crucial inequality to investigate the extension of smooth density provided that the first order Lie brackets span the whole Euclidean space. From the Stochastic Partial differential equations side, Ferrario and Zanella reviewed the ergodicity of stochastic 2D Navier-Stokes equations on a bounded smooth domain developed in the past 30 years. Some recent results on the well-posedness, the existence and uniqueness of invariant probability measures, and the algebraic or exponential ergodicity, for both additive noise and multiplicative noise respectively are summarized. Zhang, Xiong and Zhang concerned the optimal control problem. The maximum principle has been developed with cost functional is teneral risk measure instead of an expectation by using a variational method. Bao and Hao concerned the stochastic algorithm problem by passing the Malliavin calculus and, Milstein-type schemes are constructed via the functional Itô calculus for a range of functional stochastic differential equations. Guo and Wu surveyed the recent work on stochastic fractional diffusions driven by space-time white noise. This will form an important class of SPDEs, which includes many well-studied SPDEs such as stochastic heat equations. All the six papers were rigorously refereed and edited under the strict journal guidelines and policy that apply to regular issues of Communications on Mathematical Analysis and Applications. We hereby express our sincere gratitude to the authors and reviewers who contributed greatly to the success of this special issue. It is hoped that this issue will contribute to the already accelerating expansion of the very hot subject of stochastic analysis.

The special issue editors thank Professor Tong Yang and Professor Renjun Duan, the editors of CMMA, for their kind invitation and arrangement.

Panpan Ren
City University of Hong Kong, Hong Kong SAR, China,
panparen@cityu.edu.hk

Jiang-Lun Wu
Beijing Normal-Hong Kong Baptist University, Zhuhai, China,
jianglunwu@uic.edu.cn