

Normalized Ground State Solution to a Mixed Schrödinger System in a Plane

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Abstract. In this paper, we establish the existence of positive ground state solutions for a class of mixed Schrödinger systems with concave-convex nonlinearities in \mathbb{R}^2 , that is

$$\begin{cases} -\partial_{xx}u + (-\Delta)_y^s u + \lambda_1 u = \mu_1 u^{p-1} + \beta r_1 u^{r_1-1} v^{r_2}, \\ -\partial_{xx}v + (-\Delta)_y^s v + \lambda_2 v = \mu_2 v^{q-1} + \beta r_2 u^{r_1} v^{r_2-1} \end{cases}$$

subject to the L^2 -norm constraints

$$\int_{\mathbb{R}^2} u^2 dx dy = a, \quad \int_{\mathbb{R}^2} v^2 dx dy = b,$$

where $(x, y) \in \mathbb{R}^2$, $u, v \geq 0$, $s \in (1/2, 1)$, $\mu_1, \mu_2, \beta > 0$, $r_1, r_2 > 1$, the prescribed masses $a, b > 0$, and the parameters λ_1, λ_2 appear as Lagrange multipliers. Moreover, the exponents $p, q, r_1 + r_2$ satisfy $2(1+3s)/(1+s) < p, q, r_1 + r_2 < 2_s$, where $2_s = 2(1+s)/(1-s)$. To obtain our main existence results, we employ variational techniques such as the mountain pass theorem, the Pohozaev manifold, Steiner rearrangement, and others, consolidating the works [Jeanjean *et al.*, *Nonlinear Differ. Equ. Appl.* 31 (2024)].

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1 Introduction

In recent years, nonlinear fractional Schrödinger equations have attracted considerable attention due to their wide-ranging applications in fields such as nonlinear optics, ecology, biology, and fractional quantum mechanics. Among these, the mixed Schrödinger operator

$$\mathcal{L} = -\partial_{xx} + (-\Delta)_y^s$$

has emerged as a critical tool in understanding anisotropic diffusion phenomena and other complex systems. For more details about operator \mathcal{L} , we refer to [5]. The work by Esfahani and Esfahani [5] provides a foundational framework for this study by analyzing equations of the form

$$w + (-\Delta)_x^s w + (-\Delta)_y w = f(w), \quad (x, y) \in \mathbb{R}^n \times \mathbb{R}^m, \quad (1.1)$$

where $s \in (0, 1)$ and $m, n \in \mathbb{N}$. Here, the fractional Laplacian $(-\Delta)_x^s$, defined (up to a normalizing constant $C_{n,s}$) as

$$(-\Delta)_x^s w(x, y) = C_{n,s} \text{P.V.} \int_{\mathbb{R}^n} \frac{w(x, y) - w(z, y)}{|x - z|^{n+2s}} dz,$$

describes fractional diffusion in the x -direction, while the operator $(-\Delta)_y w$ accounts for classical Laplacian effects in the y -direction. The nonlinearity $f \in C^1(\mathbb{R}, \mathbb{R})$ is assumed to satisfy subcritical growth conditions, without Ambrosetti-Rabinowitz condition.

The operator \mathcal{L} has significant implications in modeling anisotropic diffusion, particularly in stochastic processes such as Brownian motion and Lévy-Itô processes, where directional sensitivity plays a vital role. These systems exhibit diffusion behavior that is not uniform in all directions, providing a mathematical representation of real-world scenarios involving non-isotropic movement.

Additionally, fractional operators are essential in describing systems with long-range interactions. For instance, the fractional Laplacian $(-\partial_{yy})^s$, defined via its Fourier transform as

$$\mathcal{F}(D_y^{2s} u)(\xi) = |\xi_2|^{2s} \mathcal{F}(u)(\xi),$$