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Two-Dimensional Steady Supersonic Relativistic Euler Flows Past Lipschitz Wedges

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Dedicated to Professor Gui-Qiang G. Chen on the occasion of his 60th birthday

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Abstract. This paper is a review of wedge problems for supersonic relativistic Euler flows. We are mainly concerned with two-dimensional compressible supersonic relativistic Euler flows past Lipschitz wedges, sharp corners, or bending wedges from mathematical point of view. When the vertex angle of the upstream flow is less than the critical angle, a shock wave is generated from the wedge vertex. If the vertex angle is larger than π and the angle of the flow is larger than the critical value, then a rarefaction wave will appear. In this paper, we employ modified wave front tracking method to establish the structural stability of such wave patterns under some small perturbations of both the upcoming supersonic flow and the tangent slope of the boundary. It is an initial-boundary value problem for two-dimensional steady compressible relativistic Euler system. Moreover, we study global non-relativistic limits of entropy solutions for relativistic Euler flows as well as the asymptotic behavior of the solutions at the infinity. In particular, we demonstrate the basic properties of nonlinear waves for the two-dimensional steady supersonic relativistic Euler flows, especially, the geometric structures of shock polar and rarefaction wave.

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Key words: Steady supersonic relativistic Euler flow, shock wave, rarefaction wave, wave front tracking method, interaction of waves, asymptotic behavior.

1 Introduction

In astrophysics, plasma physics, and nuclear physics, the macroscopic velocity of the fluid or the microscopic particle speed is close to the light speed, the relativistic effect has to be taken into consideration. The two-dimensional steady relativistic Euler equations consisting of conservation laws of momentum and energy, read as

$$\begin{cases} \nabla_{\boldsymbol{x}} \cdot \left(\dfrac{(\rho + p/c^2)\boldsymbol{u} \otimes \boldsymbol{u}}{1 - |\boldsymbol{u}|^2/c^2} \right) + \nabla_{\boldsymbol{x}} p = 0, \\ \nabla_{\boldsymbol{x}} \cdot \left(\dfrac{(\rho + p/c^2)\boldsymbol{u}}{1 - |\boldsymbol{u}|^2/c^2} \right) = 0, \end{cases} \tag{1.1}$$

where $\boldsymbol{x} = (x,y)$ is the space variable, ρ and $\boldsymbol{u} = (u,v)$ stand for the mass-energy density and velocity of the flow, respectively, p is the pressure, and c is the light speed. For polytropic gas, the state equation is given by

$$p(\rho) = \rho^{\gamma}, \quad \gamma > 1.$$

Note that $a := \sqrt{p_\rho(\rho)}$ is the local sound speed. The flow is supersonic if $q := \sqrt{u^2 + v^2} > a$, and subsonic if $q < a$.

One of the motivations to study system (1.1) is that the Newtonian limit of system (1.1) as $c \to +\infty$ is formally the classical steady Euler equations

$$\begin{cases} \partial_x(\rho u^2 + p) + \partial_y(\rho u v) = 0, \\ \partial_x(\rho u v) + \partial_y(\rho v^2 + p) = 0, \\ \partial_x(\rho u) + \partial_y(\rho v) = 0. \end{cases} \tag{1.2}$$

Wedge problem is a fundamental question in fluid dynamics. The mathematical study of the steady supersonic flows past wedges can be dated back to the 1940s, also see the references Courant and Friedrichs [16], Whitham [29] for the stability of such flows in applications. Local existence of solutions around the wedge vertex were initially studied by Gu [22], Li [25], Schaeffer [27], etc. Global existence and asymptotic behavior of potential flows were established in various backgrounds in [12–14]. Some investigations have been developed on well-posedness of nonlinear waves, such as shock, rarefaction waves and characteristic discontinuities. Zhang [30] studied two-dimensional steady supersonic irrotational potential flow past curved wedges with a small vertex angle, which is