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Hypersonic Limit for C^1 Solution of One Dimensional Isentropic Euler Equations

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Abstract. In this article, we study the hypersonic limit problem to the 1-D isentropic Euler equations. For uniformly bounded density and velocity, it can be formulated as the behavior of solution as $\gamma\to 1$. First we study and clarify the mechanism of singularity formation for two case: Only derivatives blow up when $\gamma>1$, the derivatives blow up with mass concentrates when $\gamma=1$. Then we showed as $\gamma\to 1$, the classic solutions of the isentropic Euler equations converge to the solutions of the pressureless Euler equations. We proved that u converges in C^1 and ρ converges in C^0. By a level set argument, the convergence rate is proved to be $\sqrt{\gamma-1}$ on any fixed level set. Furthermore, we show that the time that singularity forms for $\gamma>1$ converges to the time of singularity forms for $\gamma=1$.

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Key words: Compressible Euler equations, hypersonic limit, mass concentration, asymptotic behavior, convergence rate.

1 Introduction

The study of the properties of the air flows at high speeds is an important topic with many applications in physics and engineer such as hypersonic aircraft design. In mathematics, one of the important problems in this topic is: How the air

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flow changes as the Mach number M_∞ of upcoming flow tends to ∞? We call the problem the hypersonic limit problem. In fact, the problem can be formulated to the study of the behaviors of the solution of the Euler system as $M_\infty \to \infty$. It is a long history of the study of this kind of problem, which is mainly for weak solutions. Through these studies, it was found that some singular measures such as the Dirac measure δ are needed to describe the process [32]. Because of the appearance of Dirac measure, the measure solution framework is appropriate to study the problems, see [26].

In [26], the authors studied the hypersonic limit problem for the two dimensional steady compressible flow passing a straight wedge, and proved the solution converges to a Dirac measure on the straight boundary as the Mach number tends to ∞. Recently, the hypersonic limit for 1-D piston problem in the sense of measure solution is considered in [25], the hypersonic limit for the uniform supersonic flow passing through a straight circular cone in the sense of measure solution is considered in [19]. Another related problem, the rigorous validation of the hypersonic similarity is systematically established through $BV\cap L^1$ frame in [4,5,16,17].

In this paper, the hypersonic limit connects two models, the isentropic Euler equations and the pressureless equations. Thus, we need to introduce some basic properties of these two equations. The one-dimensional isentropic compressible Euler equations is

$$\begin{cases} \rho_t+(\rho u)_x=0, & \text{(1.1a)}\\ (\rho u)_t+(\rho u^2+P)_x=0, & \text{(1.1b)} \end{cases}$$

where ρ is the density of the fluid, u is the velocity, $P=(\gamma-1)\rho^\gamma$ is the pressure of the fluid. The initial condition is given by

$$\big(\rho(0,x),u(0,x)\big)=\big(\rho_0(x),u_0(x)\big)\in\big(C^1(\mathbb{R})\big)^2. \tag{1.2}$$

At $x=\infty$, we denote the free stream as

$$\lim_{x\to\pm\infty}\big(\rho_0(x),u_0(x)\big)=(\rho_\infty,u_\infty). \tag{1.3}$$

Furthermore, we require that there exists a positive number C_1 such that

$$C_1^{-1}<\rho_0(x)<C_1. \tag{1.4}$$

The sound speed c is defined as $c=\sqrt{\gamma P/\rho}$. Define the Mach number $M=|u|/c$. The flow is called supersonic when $M>1$. The Mach number M_∞ of the free stream is given by

$$\frac{1}{M_\infty^2}=\frac{\gamma(\gamma-1)\rho_\infty^{\gamma-1}}{u_\infty^2}, \tag{1.5}$$