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Error Bounds for Linear Complementarity Problem of SDD_k Matrices

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Abstract. SDD_k matrices are a subclass of the nonsingular H-matrices. The infinity norm of the inverse for SDD_k matrices has been given. In the paper, we utilize this result in the context of the linear complementarity problem, and the error bounds of the linear complementarity problem for SDD_k matrices are obtained. By the relationship between the SDD matrices and the SDD_k matrices, we further obtain the error bounds of the linear complementarity problem for SDD matrices. In addition, it is proved that the bounds presented in this paper are sharper than the well-known bounds under some conditions. Finally, numerical examples are provided to demonstrate the effectiveness of our results.

AMS subject classifications: 15A39, 15A60, 90C33

Key words: Linear complementarity problem, error bound, SDD_k matrices, SDD matrices.

1 Introduction

Let n be a positive integer, $N=\{1,2,\dots,n\}$, and $\mathbb{R}^{n\times n}$ ($\mathbb{C}^{n\times n}$) be the set for all real (complex) matrices of order n. The linear complementarity problem of matrix $\boldsymbol{M}$, denoted by $LCP(\boldsymbol{M},\boldsymbol{q})$, is to find a vector $\boldsymbol{x}\in\mathbb{R}^n$ such that

$$\boldsymbol{x}\geq\boldsymbol{0},\quad \boldsymbol{M}\boldsymbol{x}+\boldsymbol{q}\geq\boldsymbol{0},\quad \boldsymbol{x}^T(\boldsymbol{M}\boldsymbol{x}+\boldsymbol{q})=\boldsymbol{0},$$

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or to prove that such a vector $\boldsymbol{x}\in\mathbb{R}^n$ does not exist, where $\boldsymbol{M}=(m_{ij})\in\mathbb{R}^{n\times n}$ and $\boldsymbol{q}\in\mathbb{R}^n$. $LCP(\boldsymbol{M},\boldsymbol{q})$ has been rapidly expanded to many fields such as economics, engineering, game theory, etc. Further, the $LCP(\boldsymbol{M},\boldsymbol{q})$ has a wide range of applications in problems such as quadratic programming, Nash equilibrium of a bimatrix game and the free boundary problem in fluid mechanics, see [1,3,12] for details.

The $LCP(\boldsymbol{M},\boldsymbol{q})$ has a unique solution for any $\boldsymbol{q}\in\mathbb{R}^n$ if and only if $\boldsymbol{M}$ is a P-matrix [3]. Chen *et al.* [2] gave an error bound for $LCP(\boldsymbol{M},\boldsymbol{q})$ with P-matrix $\boldsymbol{M}$

$$\|\boldsymbol{x}-\boldsymbol{x}^*\|_\infty\le\max_{d\in[0,1]^n}\|(\boldsymbol{I}-\boldsymbol{D}+\boldsymbol{D}\boldsymbol{M})^{-1}\|_\infty\|\boldsymbol{r}(\boldsymbol{x})\|_\infty,$$

where $\boldsymbol{x}^*$ is the exact solution of $LCP(\boldsymbol{M},\boldsymbol{q})$, $\boldsymbol{r}(\boldsymbol{x})=\min\{\boldsymbol{x},\boldsymbol{M}\boldsymbol{x}+\boldsymbol{q}\}$, $\boldsymbol{D}=\mathrm{diag}(d_1,d_2,\cdots,d_n)$, with $0\le d_i\le 1$ and the min operator $\boldsymbol{r}(\boldsymbol{x})$ denotes the componentwise minimum of two vectors. While the estimation of $\|\boldsymbol{r}(\boldsymbol{x})\|_\infty$ is relatively simple, the computation of $\max_{d\in[0,1]^n}\|(\boldsymbol{I}-\boldsymbol{D}+\boldsymbol{D}\boldsymbol{M})^{-1}\|_\infty$ becomes difficult for large or complex matrices, this motivates us to focus on estimating the $\max_{d\in[0,1]^n}\|(\boldsymbol{I}-\boldsymbol{D}+\boldsymbol{D}\boldsymbol{M})^{-1}\|_\infty$. Real H-matrices $\boldsymbol{A}$ with positive diagonal entries are a subclass of P-matrices, so the error bound of $LCP(\boldsymbol{A},\boldsymbol{q})$ can be calculated by using the bound [2, (2.4)].

In recent years, many scholars have estimated the error bounds of the linear complementarity problem for different matrices through the infinity norm of inverse for matrices, and have continuously improved them, for example, Nekrasov matrices [9], S-Nekrasov matrices [7], QN-matrices [10], S-QN-matrices [11], Ostrowski matrices [4] and DZT matrices [16]. For SDD_k matrices, the upper bounds of the infinity norm for the matrices inverse have been given in [14].

In this paper, we mainly focus on the error bounds of the linear complementarity problem related to SDD_k matrices. Based on the fact that SDD_k matrices are a subclass of nonsingular H-matrices, the error bounds of the linear complementarity problem for SDD_k matrices are obtained. In particular, the error bounds for the linear complementarity problem of SDD_k matrices without the need for any parameters are presented. Furthermore, based on the fact that the class of SDD matrices is a subclass of SDD_k matrices, the error bounds for SDD matrices with and without parameters are obtained. Notably, we prove that the bounds obtained in this paper are sharper than the well-known bounds in [8] under some conditions.

The rest of the paper is organized as follows. In Section 2, the relevant definitions and theorems used in the following text are given. In Section 3, the main results of this paper are presented. We give the error bounds for the linear complementarity problem of SDD_k and SDD matrices. In addition, it is proved that