Transport of Contaminant Release by Three Different Sources: A Medium with Complex Geometry

Abdon Atangana[1,2,*]

[1] *Institute for Groundwater Studies, Faculty of Natural and Agricultural Sciences, University of the Free State, South Africa.*
[2] *IT4Innovations, VSB – Technical University of Ostrava, 17. listopadu 2172/15, 708 00 Ostrava-Poruba, Czech Republic.*

Abstract. This study develops a mathematical model for contaminant transport from two industrial sources and one natural source through a heterogeneous aquifer featuring fracture networks with variable apertures, faults of heterogeneous width, and a complex rock matrix. Key innovations include: (1) A nonlinear Darcy's law with concentration-dependent conductivity; (2) Three coupled advection-dispersion equations with nonlinear dispersion and reaction terms for multispecies interactions; (3) Seasonal recharge dynamics integrated into flow-transport coupling. Non-dimensional analysis reveals advection-dominated regimes governed by Péclet and Damköhler numbers. Bifurcation analysis identifies stability thresholds for ternary chemical reactions. Numerical solutions via the Crank-Nicolson scheme demonstrate fracture-controlled contaminant pathways and recharge-modulated plume evolution. The framework provides critical insights for pollution management in geologically complex aquifers.

1 Introduction

It is a well-known fact that pollution is a threat to the environment and human beings [8, 12]. Pollutions are generated naturally and artificially due to various human activities [7, 11]. For example, agriculture, industry, domestic, and others. Some of these

---

Corresponding author. Email address: `AtanganaA@ufs.ac.za` (A. Atangana)

pollutants are released into nature and thus come into contact with the geological formation where the subsurface water is found [3, 6]. This form of pollution is usually referred to as groundwater pollution. In general, groundwater pollution occurs when pollutants are deposited in the ground above an aquifer; thus, through recharge processes, pollution makes its way to groundwater. It is also worth noting that pollution can occur naturally, which is sometimes considered unwanted constituents, impurities, and contaminants in groundwater. It is worth noting that using polluted groundwater leads to severe public health issues, such as poisoning or the spread of waterborne diseases [4, 5]. In many water-scarce countries, groundwater serves as a source of freshwater that can be used for different purposes. For example, many farmers use groundwater for agriculture, many people use it for domestic purposes, and others for industry. Pollution is transported through the geological formation when in contact with the groundwater. The transport of pollution throughout aquifers has attracted the attention of many researchers in the last decades [2, 13]. The Darcy law [14, 15], groundwater flow equations [9, 10], and the advection-dispersion equations [1] have been used intensively in the last decades to understand the transport of pollution through aquifers. In this paper, we will consider a scenario where three sources of pollution contaminate an aquifer. The two sources are from two different chemical companies, and one is a natural source. The aquifer is highly heterogeneous, composed of faults, fractures, and a complex rock matrix. The aim is to derive a transport equation that could be used to explain the transport of this pollution via the aquifer.

The paper will be structured as follows. In the Section 2, we will provide a detailed derivation of this model. In Section 3, we will present a detailed analysis of the dimensions. In Section 4, we will present a bifurcation analysis of the interaction of the three species. In Section 5, a Fourier harmonic analysis will be presented with some graphics reflecting the effect of periodic changes in recharge, and finally, numerical analysis will be presented, as well as numerical simulations.

2   Mathematical model

In this section, we shall present a mathematical model to replicate processes described above. The Cartesian coordinate will be considered here such that $\Lambda \subset \mathbb{R}^2$ will be the domain of the selected aquifer [9, 10], which will be described by (x, y) and t. Since there are three sources of pollution, let $C_c^1(x, y, t)$ be the concentration of pollution released by Company 1. $C_c^2(x, y, t)$ the concentration of pollution released by Company 2 and $C_n(x, y, t)$ the concentration. The fractures and faults can be added, noting that these regions are described by an area with variable permeability $K(x, y)$. Due to the complexity of the media, it is possible to observe nonlinear flow, therefore, to include in the mathematical formulation this aspect. We adopt a nonlinear Darcy's law [14, 15]

$$q = -K(C)\nabla h. \tag{2.1}$$