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A Novel Structure-Preserving Scheme for Three-Dimensional Maxwell's Equations

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Abstract. In this paper, a novel structure-preserving scheme is proposed for solving the three-dimensional Maxwell's equations. The proposed scheme can preserve all of the desired structures of the Maxwell's equations numerically, including five energy conservation laws, two divergence-free fields, three momentum conservation laws and a symplectic conservation law. Firstly, the spatial derivatives of the Maxwell's equations are approximated with Fourier pseudo-spectral methods. The resulting ordinary differential equations are cast into a canonical Hamiltonian system. Then, the fully discrete structure-preserving scheme is derived by integrating the Hamiltonian system using a sixth order average vector field method. Subsequently, an optimal error estimate is established based on the energy method, which demonstrates that the proposed scheme is of sixth order accuracy in time and spectral accuracy in space in the discrete L^2-norm. The constant in the error estimate is proved to be only $\mathcal{O}(T)$, where $T>0$ is the time period. Furthermore, its numerical dispersion relation is analyzed in detail, and a customized fast solver is presented to efficiently solve the resulting discrete linear equations. Finally, numerical results are presented to validate our theoretical analysis.

AMS subject classifications: 65M12, 65M15, 65M70

Key words: Maxwell's equations, AVF method, structure-preserving scheme, dispersion relation, divergence preservation.

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1 Introduction

The Maxwell's equations describe the propagation and scattering of electromagnetic waves and have a wide variety of applications in science and engineering, including microwave circuits, radio-frequency, antennas, aircraft radar, integrated optical circuits, wireless engineering, etc. Various applications stimulate the investigation of constructing of efficient numerical methods for the Maxwell's equations. A well-known numerical method in computational electromagnetics is the finite-difference time domain (FDTD) method, which was first introduced by Yee in [39] and further developed and analyzed in [23, 34]. However, the Yee-based FDTD method is only conditionally stable so it may require very small temporal step-size and suffer from impractical computational cost for long time computations.

Based on the basic rule that numerical methods should preserve the intrinsic properties of the original problems as much as possible, Feng [13] first presented the concept of symplectic schemes for Hamiltonian systems and furthered the structure-preserving methods for the general conservative dynamical systems. Due to their superior properties in long time numerical computations over traditional numerical methods, structure-preserving methods have been proved to be very powerful in scientific research (e.g. see [14,16,25,37] and references therein). In the past few decades, the symplectic schemes have gained remarkable success in solving the Maxwell's equations (e.g. see [4, 17, 19, 27, 29, 31–33, 41] and references therein). In addition to the symplectic conservation law, the Maxwell's equations also admit five energy conservation laws, three momentum conservation laws and two divergence-free fields, which are very important invariants for long time propagations of the electromagnetic waves [34]. Chen *et al.* [10] proposed an energy-conserved splitting scheme for two-dimensional (2D) Maxwell's equations in an isotropic, lossless and sourceless medium. Further analysis in three-dimensional (3D) case was investigated in [11]. Other energy-conserved splitting schemes can be found in [2, 18, 19, 21]. However, the energy-conserved splitting schemes have only been strictly proven to preserve the energy conservation laws [10, 11]. Motivated by the basic idea of structure-preserving methods, it is valuable to expect that the numerical schemes that preserve multiple conservation laws of the Maxwell's equations will produce richer information on the discrete systems. Furthermore, most of the existing energy-conserved splitting schemes have only second order accuracy in both time and space to the best of our knowledge. The low order schemes are usually effective for geometries of moderate electrical size, but, for computing large scale problems, for problems requiring long-time integration, or for problems of wave propagations over longer distances [21]. The goal of this paper is to develop a novel sixth-order structure-preserving scheme (6th-SPS) for the Maxwell's equations, which enjoys the following advantages:

- is unconditionally stable and non-dissipative (numerical dispersion analysis),
- is symmetric and can preserve the discrete version of symplecticity, five energy conservation laws, three momentum conservation laws, as well as two discrete divergence-free fields.