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An Inexact Framework of the Newton-Based Matrix Splitting Iterative Method for the Generalized Absolute Value Equation

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Abstract. An inexact framework of the Newton-based matrix splitting (INMS) iterative method is developed to solve the generalized absolute value equation, whose exact version was proposed by Zhou, Wu and Li [J. Comput. Appl. Math., 394, 2021]. Global linear convergence of the INMS iterative method is investigated in detail. Some numerical results are given to show the superiority of the INMS iterative method.

AMS subject classifications: 65F10, 65H10, 90C30

Key words: Generalized absolute value equation, Newton-based matrix splitting method, inexact iterative method, linear convergence.

1 Introduction

In this paper, we concentrate on the solution of the generalized absolute value equation (GAVE)

$$Ax-B|x|-b=0, \tag{1.1}$$

where $A,B\in\mathbb{R}^{n\times n}$, $b\in\mathbb{R}^n$ are known and $|x|=(|x_1|,|x_2|,\cdots,|x_n|)^\top$ denotes the component-wise absolute value of the unknown vector $x=(x_1,x_2,\cdots,x_n)^\top\in\mathbb{R}^n$. GAVE (1.1) is first

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introduced in [33] and further investigated in [12,21,30,39] and the references therein. In the special case of $B=I$ or B is nonsingular, GAVE (1.1) can be reduced to the absolute value equation (AVE)

$$Ax-|x|-b=0. \tag{1.2}$$

The main significance of GAVE (1.1) and AVE (1.2) arises from the fact that they have many applications in optimization fields such as linear programming problems, bimatrix games, mixed integer programming, complementarity problem, quadratic programming and others, see, e.g. [1,21,24,25,30,32] and references therein. Particularly, if $B=0$, then GAVE (1.1) reduces to the linear system $Ax=b$, which plays a significant role in scientific computation. Recently, Ling *et al.* [20] considered the tensor absolute value equation, which is an interesting generalization of GAVE (1.1) and AVE (1.2) in the context of multilinear systems.

In recent years, GAVE (1.1) and AVE (1.2) have been extensively investigated, and the research efforts have mainly centered on two aspects: providing theoretical analysis and exploring efficient numerical methods. On the theoretical side, many studies have focused on the equivalent reformulations of GAVE (1.1) or AVE (1.2), and detected the existence and nonexistence of solutions, see, e.g. [21,25,28,30,37–39] and the references therein. Especially, it has been proved in [21] that determining the existence of a solution to general GAVE (1.1) or AVE (1.2) is NP-hard. Furthermore, in [30], it has been shown that checking whether GAVE (1.1) or AVE (1.2) has a unique solution or multiple solutions is NP-complete. On the numerical side, it focuses on exploring efficient numerical algorithms for solving GAVE (1.1) and AVE (1.2). For example, the Newton-type methods [2, 3, 23, 36, 41], the neural network approaches [4, 26, 27], the SOR-like iterations [6, 10, 16, 17], the concave minimization methods [1, 21, 22, 42], the conjugate gradient method [31] and others, see, e.g. [7,9,14,15,35,40] and the references therein. In the following, we go in for a closer look on some Newton-type methods.

By considering GAVE (1.1) as a system of nonlinear equations

$$F(x)=0 \quad \text{with} \quad F(x):=Ax-B|x|-b, \tag{1.3}$$

some Newton-type algorithms for solving nonsmooth equations are developed to find a solution of GAVE (1.1) and AVE (1.2). Mangasarian [23] utilized the generalized Jacobian $\partial|x|$ of $|x|$ based on a subgradient of its components and directly proposed the generalized Newton (GN) iterative method to solve AVE (1.2). Hu *et al.* [13] then extended the GN iteration to solve GAVE (1.1). As the Jacobian matrix of the GN iteration is changed at each iterative step, it is undesirable for solving large-scale problems, especially if the Jacobian matrix is ill-conditioned. To overcome this shortcoming, modified Newton-type (MN) iterative methods were developed for solving GAVE (1.1) [36]. Following that, a new MN (NMN) iterative method was proposed in [19], which is more balanced than the MN method. Subsequently, a more general Newton-based matrix splitting (NMS) method was established to solve GAVE (1.1) [43]. As shown in [43], by choosing suitable matrix splitting, the NMS method can include the Picard method [34] and the MN