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ℓ_1DecNet+: A New Architecture Framework by ℓ_1 Decomposition and Iteration Unfolding for Sparse Feature Segmentation

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Abstract. ℓ_1 based sparse regularization plays a central role in compressive sensing and image processing. In this paper, we propose ℓ_1DecNet, as an unfolded network derived from a variational decomposition model, which incorporates ℓ_1 related sparse regularizations and is solved by a non-standard scaled alternating direction method of multipliers. ℓ_1DecNet effectively separates a spatially sparse feature and a learned spatially dense feature from an input image, and thus helps the subsequent spatially sparse feature related operations. Based on this, we develop ℓ_1DecNet+, a learnable architecture framework consisting of our ℓ_1DecNet and a segmentation module which operates over extracted sparse features instead of original images. This architecture combines well the benefits of mathematical modeling and data-driven approaches. To our best knowledge, this is the first study to incorporate mathematical image prior into feature extraction in segmentation network structures. Moreover, our ℓ_1DecNet+ framework can be easily extended to 3D case. We evaluate the effectiveness of ℓ_1DecNet+ on two commonly encountered sparse segmentation tasks: retinal vessel segmentation in medical image processing and pavement crack detection in industrial abnormality identification. Experimental results on different datasets demonstrate that, our ℓ_1DecNet+ architecture with various lightweight segmentation modules can achieve equal or better performance than their enlarged versions respectively. This leads to especially practical advantages on resource-limited devices.

AMS subject classifications: 94A08, 68T01, 68U10

Key words: Variational model, ℓ_1 regularization, ℓ_1 decomposition, ADMM, deep unfolding, sparse feature extraction, sparse feature segmentation.

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1 Introduction

Image segmentation is one of the most important middle-level vision tasks, which bridges low-level vision operations and high-level vision applications. So far, people developed lots of conventional methods and learning-based methods. In whatever method, an accurate suitable feature description of each object content is a key. In conventional segmentation methods, object features are explicitly modeled, such as a constant or polynomial intensity function. In learning-based methods, object features are implicitly learned in neural networks and usually lack of interpretability.

In this paper, we consider the problem of sparse feature extraction and segmentation from an input image. This problem raises in diverse applications like retinal vessel segmentation and crack anomaly detection. To solve this problem, we assume an input image be an addition of a sparse feature and a hard-to-describe dense feature (background), with possibly some measurement noise. Our idea is to combine mathematical modeling and machine learning spirits, by introducing an ℓ_1 decomposition model and using deep unfolding strategy.

Based on the powerful ℓ_1 regularization, also even called "modern least squares", we propose a decomposition minimization model, which decomposes an input image to two meaningful components, one as the sparse feature and the other as the dense feature (background). The sparse feature component is characterized by an ℓ_1 regularization, while the hard-to-describe background component is characterized by an ℓ_1 regularization composed by some sparsifying linear transformations which will be learned from training data. After deriving a non-standard scaled alternating direction method of multiplier (ADMM) solver for this composite optimization problem, we unfold the iterative scheme to construct a deep neural network to build our ℓ_1DecNet. Then an architecture framework named ℓ_1DecNet+ is developed, by connecting our ℓ_1DecNet and an any segmentation module; see Fig. 1. The ℓ_1DecNet extracts a sparse feature well from an input image and deliver the feature to the subsequent segmentation module to finalize the segmentation task. As can be seen, sparsity priors (with or without learnable linear transformations) for feature descriptions are embedded into ℓ_1DecNet+. Therefore, it combines well mathematical modeling and data-driven spirits. To train ℓ_1DecNet+, we use adaptive moment estimation (ADAM) to minimize a loss function consisting of a segmentation loss and a feature loss.

We conduct our experiments of ℓ_1DecNet+ on three datasets DRIVE, CHASE_DB1 and CRACK for sparse feature segmentation. We construct six ℓ_1DecNet+ architectures by using six popular lightweight segmentation networks as the segmentation modules for our experiments. Tests and comparisons show that our ℓ_1DecNet+ has the flexibility on the choice of segmentation module. While achieving an equal or better performance of the compared large network related to the used segmentation module, our ℓ_1DecNet+ architecture uses much less learnable parameters and much less storage occupation for network weights.

Our contributions are as follows: