# Invasion Dynamics of Alien Mussels: A Mussel-Algae Model with Time Delay in Filter Feeding and Environmental Variability

Wenxu Ning[1], Jie Lou[2], Litao Han[3] and Sanling Yuan[1,*]

[1] *College of Science, University of Shanghai for Science and Technology, Shanghai 200093, P.R. China.*
[2] *Department of Mathematics, Shanghai University, Shanghai 200444, P.R. China.*
[3] *School of Mathematics, Renmin University of China, Beijing 100872, P.R. China.*

**Abstract.** The dynamic behavior of alien mussels interacting with algae after arriving in a new environment has long been a focus of invasion ecology research. This paper extends and analyzes a classical mussel-algae model by incorporating a time delay in mussel filter feeding and accounting for environmental variability. We theoretically study the stochastic dynamics, including the global existence and uniqueness of the positive solution, the existence of a unique stationary distribution, and mussel extinction, using tools from stochastic analysis. Furthermore, we derive an explicit expression for the probability density function around the quasi-stable equilibrium by solving the corresponding Fokker-Planck equation. Our theoretical and numerical results indicate that: (a) larger environmental disturbances or artificial removal can effectively prevent the survival of alien mussels in novel habitats, (b) a decreased filter feeding rate leads to an accelerated extinction rate of mussels, and (c) an increased consumption constant $c$ decelerates the transition rate of mussels from the initial state to the extinction state, as analyzed through the mean first passage time of mussels. These findings highlight the complex interaction between intrinsic and extrinsic factors in influencing the invasion dynamics of alien mussels.

**AMS subject classifications**: 34F05, 92B05, 37H10

**Key words**: Stochastic mussel-algae model, infinite distributed delay, stationary distribution, extinction, density function.

## 1 Introduction

Globally, biological invasions pose a significant threat to biodiversity and ecosystem function [38]. Invasive species impact ecosystems through competitive interactions, predation on native species, habitat alterations, nutrient cycles, and energy budgets [2, 21].

---

*Corresponding author. Email address:* `sanling@usst.edu.cn` (S. L. Yuan),

The zebra mussel exemplifies a highly successful invasive species, distinguished by its prolific reproductive capacity, rapid growth rate, broad tolerance to diverse physical and chemical conditions, and its ability to establish advantageous interactions with native species [26]. One major consequence of zebra mussels invasion is biofouling, which obstructs water intake structures, such as pipes and screens, thereby impairing pumping efficiency for power and water treatment facilities, and incurring costs for industries, businesses, and communities. In addition, recreation-based industries and activities have also been significantly affected, as docks, buoys, boats, and beaches have been extensively colonized [25].

Zebra mussels also disrupt the ecosystems they invade. They can shift lakes from a turbid, phytoplankton-dominated state to a clear, macrophyte-dominated state through filter-feeding, potentially leading to increased competition, decreased survival, and reduced biomass of planktivorous fish [9, 31]. After filter-feeding, the mussels release unwanted inorganic particles as pseudofeces. The decomposition of this waste consumes oxygen, lowers pH, and produces toxic byproducts. Additionally, biomagnification of organic pollutants can occur as pseudofeces are passed up the food chain [3].

Control and prevention of macrofouling caused by zebra mussels have become major concerns for managers of various water delivery systems. Currently, there are several management techniques to control invasive mussels, including mechanical removal [35], oxidizing biocides [42], and biological control [37]. The ineffectiveness of traditional control strategies and their potential environmental harm necessitate a shift toward prevention as the optimal approach for managing invasive species [1]. Consequently, a thorough investigation into the population dynamics of zebra mussels can enhance our understanding of their invasion conditions. Moreover, manual counting remains the primary method for quantifying the population size of benthic fauna [6], which poses challenges due to the potential magnitude of the population. In this context, some mathematical models yield interesting quantitative results [39]. Additionally, a mathematical model can provide qualitative insights into infestation dynamics, effectively aiding in the prevention process. Thus, a model considering the interaction of mussels and algae could be beneficial for evaluating invasion potential and population control strategies [15, 24].

Recent research modeling the interaction between algae and mussels has been conducted in [1, 4, 15, 18, 19, 32, 34]. Koppel *et al.* [15], first proposed a mathematical model of algae and mussels based on a distinct mechanism: reduced mussel mortality at high mussel density, leading to intraspecific facilitation. Their study emphasizes that self-organization may influence resource flow through ecosystems, thereby affecting ecosystem functioning at larger spatial scales. Cangelosi *et al.* [4] extended the model in [15] by substituting the advective term in the algae equation with a lateral diffusive term for mussel biomass density and algae concentration in a quiescent marine layer over an unbounded soft sedimentary substrate, predicting periodic mussel bed patterns in this context. Silva *et al.* [32] developed a three-dimensional mathematical model incorporating the relationships among larvae, mussels, and algae, integrating hydrodynamics and population dynamics. They quantified the population of the invasive golden mussel in