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A Two-Patch Epidemic Model with Prevalence-Dependent Contact Patterns and Migration Rates

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Dedicated to Prof. Ma Zhien on the occasion of his 90th birthday,
with deep gratitude for his thirty-year friendship, his kindness,
and the inspiration drawn from his lectures and papers.

Abstract. We consider a two-patch Susceptible-Infected-Recovered epidemic model that incorporates awareness-driven behavioural changes in both contact and mobility patterns. Individuals modify their behaviour in response to a perceived risk of infection, which is modelled through two awareness variables that depend either on the current or past disease prevalence in each patch. We qualitatively analyse the model through stability and bifurcation theory and derive threshold conditions that determine the existence and stability of the biologically relevant equilibria. We find that awareness-induced behavioural changes in contact and mobility can destabilise mixed equilibria – where the disease persists in one patch only – and contribute to the emergence of stable co-endemic states. When awareness depends on past epidemic values, the stability analysis shows that mixed equilibria may lose stability via Hopf bifurcations, depending on the sign of some awareness-related parameters. Finally, the impact of the behaviour-related parameters on the epidemic dynamics is investigated through numerical simulations.

AMS subject classifications: 92D30, 34C60

Key words: Mathematical epidemiology, metapopulation, SIR patch model, nonconstant dispersal, awareness.

1 Introduction

Travels of individuals between different geographical regions play a major role in the dynamics of epidemics [19,35,42]. In a highly interconnected world, human mobility as-

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sociated with trade, migration, or tourism facilitates the spread of pathogens across heterogeneous environments [56]. Massive and continuous travels between different cities and countries enhanced the spread of the 2003 SARS epidemic [37,46], the 2009 H1N1 influenza pandemic [33,50] and also the global spread of COVID-19 [16,35]. Mathematical modelling, when combined with medical and public health tools, can help to improve the understanding of epidemics in different geographical regions and the development of adequate control strategies [35,42].

Spatial movements of individuals can be modelled in several ways. One approach treats space as a continuous variable, leading to partial differential equation models [1,38,41,44]. In this context, individuals are assumed to move from a given point to their direct neighbourhood. This assumption, suitable for describing phenomena in which the movement of individuals occurs locally and continuously in space, does not fit phenomena involving long-distance movements (e.g. air travel, trade routes, or rapid migrations). Alternatively, modelling space as a discrete variable leads to the so-called metapopulation models, compartmental models in which space is divided into a finite number of patches. A patch can denote a city or a geographic area, and the movements of individuals in space are modelled as dispersal between distinct patches [3,4,38].

Since the 1960s, many researchers have focused on formulating adequate metapopulation models for infectious diseases, and detailed reviews can be found in [3,4,7,37,38] and references therein. In 2003, Wang and Mulone [52] formulated a two-patch susceptible-infected-susceptible (SIS) compartmental model with standard incidence and interpatch movements allowed for both susceptible and infectious individuals. The work addresses the issue of disease persistence, identifying the unity as a sharp threshold of the basic reproduction number that determines whether the disease dies out or persists within the population. Jin and Wang [32], Wang and Zhao [53] considered an n-patch SIS model with susceptible and infectious dispersal, and analysed the effects of migrations on the dynamics of the epidemics. Among the main results, they found that for specific birth functions, population dispersal can lead to the coexistence of multiple endemic equilibria or even multi-stable endemic equilibria if the basic reproduction number is greater than one. In 2010, Yang, Wu, Li and Ma [58] considered a two-patch susceptible-infected-recovered (SIR) model in which the migration of infectious individuals is neglected, as they are considered banned from travelling due to medical screening. Differently to previous works on patch models, the authors focused on the global asymptotic stability of equilibria, rather than on the persistence of the disease. Other contributions on patch models can be found in [5, 6, 8, 31], where compartmental models with more complex structures are formulated.

In recent years, increasing attention has been devoted to developing patch models in which contact or dispersal rates are no longer assumed constant, but are modulated in response to the state of the epidemic. This modelling approach is motivated by the growing influence of social media and digital communication, which can rapidly shape public awareness, risk perception, and individual behaviour. The dissemination of information by public health authorities, together with media-driven awareness campaigns,