

# A Brucellosis Model with Different Clinical Symptoms and a Time Delay Representing Incubation Period: Global Dynamics and Data Fitting

Siyu Kang<sup>1,2,3</sup>, Rui Xu<sup>1,2,\*</sup> and Huarong Ren<sup>1,2,3</sup>

<sup>1</sup> Complex Systems Research Center, Shanxi University, Taiyuan 030006, P.R. China.

<sup>2</sup> Complex Systems and Data Science Key Laboratory of Ministry of Education, Shanxi University, Taiyuan 030006, P.R. China.

<sup>3</sup> School of Mathematics and Statistics, Shanxi University, Taiyuan 030006, P.R. China.

Received 30 May 2025; Accepted 17 August 2025

---

**Abstract.** Brucellosis is a significant zoonotic disease that has a high incidence rate in sheep, particularly in the Inner Mongolia region of China. To better investigate its transmission dynamics, this paper proposes a sheep brucellosis model incorporating acute and chronic infections, a saturated incidence rate describing environmental transmission and a time delay representing the incubation period. By constructing Lyapunov functionals and using LaSalle's invariance principle, it is shown that the global dynamics of the disease is completely determined by the basic reproduction number: If  $\mathcal{R}_0 < 1$ , the brucellosis always dies out; if  $\mathcal{R}_0 > 1$ , a unique endemic equilibrium exists and is globally asymptotically stable. Using data of sheep brucellosis cases in Inner Mongolia from 2016 to 2020, unknown parameters and the basic reproduction number ( $\mathcal{R}_0 = 1.544$ ) are estimated via the Markov chain Monte Carlo method. Numerical simulations show that reducing the transmission rate of acutely infected sheep while increasing the culling rate of symptomatic infected sheep is the most effective strategy to control the spread of brucellosis in Inner Mongolia. Notably, decreasing the proportion of acutely infected sheep or increasing acute-to-chronic conversion rate may lead to a significant short-term increase in chronically infected sheep.

**AMS subject classifications:** 34K20, 92D30

**Key words:** Brucellosis, chronic infection, basic reproduction number, global stability, data fitting.

---

## 1 Introduction

Brucellosis, also known as undulant fever, Mediterranean fever, or Malta fever, is a significant zoonotic infectious disease caused by bacteria of the genus *Brucella* [4, 33]. It has a global distribution and poses substantial threats to both public health and ani-

---

\*Corresponding author. *Email address:* xurui@sxu.edu.cn (R. Xu)

mal health. The disease is particularly prevalent in regions such as the Middle East, the Mediterranean, Mexico, and Central America [12,29,30,34]. In China, brucellosis is classified as a Category II animal epidemic disease [8]. Although the number of outbreaks and infected animals in livestock decreased by 21.75% and 16.48% respectively in 2024, the pathogen persisted and continued to spread to the southern regions in 2025, maintaining a severe disease control situation [7,38,53]. The genus *Brucella* includes multiple species, among which *Brucella abortus* and *Brucella melitensis* are the primary pathogens, capable of spreading to humans through contact or consumption of contaminated animal products [42]. This study focuses on the transmission dynamics of brucellosis in sheep populations.

Brucellosis in sheep is primarily transmitted through direct contact with infected sheep or their secretions (such as aborted fetuses, amniotic fluid, and milk). It can also be spread indirectly through contaminated feed, water, or shared equipment. The incubation period of the disease is variable, usually lasting from 1 to 3 weeks, but it can sometimes be prolonged to several months or even over a year [39,42]. Infected sheep show acute and chronic clinical symptoms. Acute infected sheep develop symptoms such as fever, abortion, and orchitis within a few weeks, which severely impact reproduction and production [41]. Chronic infected sheep, on the other hand, show more insidious symptoms such as weight loss, arthritis, or lameness, which can persist for several months or even years. *Brucella* infection not only compromises the health of sheep but also poses a threat to the livestock economy and public health.

Since the initial isolation of *Brucella* in 1886 [5], various mathematical models have been developed to study its transmission dynamics [1–3,11,13,20,27,32,36,43,44,47,48,55], but existing models still have limitations. Early studies widely used bilinear incidence to describe environmental transmission, but such models had clear limitations, as they struggled to reflect the saturation effect from bacterial concentration thresholds. To tackle this, Zhang *et al.* [46] innovatively introduced saturated incidence, which greatly boosted model authenticity and quantitatively analyzed prevention and control costs, offering key theoretical support for formulating strategies. However, this model overlooked the impact of the incubation period on brucellosis transmission. Hou and Zhang [18] attempted to represent the incubation period by introducing a time delay term. However, their research indicated that the influence of time delay on the stability of equilibria of the system was relatively limited, and it failed to consider acute and chronic infections. To address this issue, Liu *et al.* [22] further developed the susceptible-latent-acute infected-chronic infected-brucella (SLICB) model, incorporating seasonality, spatial heterogeneity, and nonlocal time delay. Their findings confirm that prolonging the incubation period and increasing the random walk rate of infected sheep can effectively prevent brucellosis from becoming endemic; moreover, considering only acute or chronic infections leads to significant deviations in the density of infected sheep and the time to reach a steady state. Notably, this model has not yet included the saturated incidence mechanism. Recently, the SEIVWS<sub>h</sub>I<sub>ah</sub>I<sub>ch</sub> model proposed by Liu *et al.* [23], although it systematically evaluated the effectiveness of various prevention and control measures such as vaccination,