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Mathematical Analysis of a Predator-Prey System with Adaptive Prey Motion

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Abstract. A mathematical model is proposed that describes the adaptive spatial movement of prey towards higher population density to reduce predation risk. The model admits the increased nonlinearity and the global existence of solutions of the system is established in Sobolev space through analytical estimates. The conditions for the Turing instability from a coexistence steady state are obtained, and sharp conditions for the asymptotical stability of the positive equilibrium in a large region are established with the help of a Lyapunov function. Numerical simulations are presented to support the theoretical results and demonstrate the versatility of spatial models.

AMS subject classifications: 92D25, 35K57

Key words: Prey aggregation, group defense, global existence, equilibrium stability, spatial pattern.

1 Introduction

A key issue in ecology is to understand how spatial distribution of population evolves with time. Mathematical models are proposed to find how the spatial patterns of popu-

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lations match those in real biological systems (see, for example, [1–3,5,8,13–15,27,31,38] and the references cited therein). Prey-predator interactions play fundamental role in ecology and their spatial evolutions affect the stability of biological system. It is shown in papers [18,22,30,36–38] that appropriate functional responses or time delays in population reaction can induce Turing instability. Further, spatial patterns driven by population movement are investigated by papers which include [10,21,26,28,32]. Essentially, prey taxis is incorporated to simulate the pursuit of predators to prey and predator-taxis is introduced to mimic the evasion of prey to predator. Interestingly, the spatial motion of predators to avoid higher prey density is introduced in [26], which leads to the emergence of spatial patterns. Notice that prey aggregation is a common and efficient way to form group defense against predator [7,11,17]. In the present paper, we incorporate the spatial motion of prey aggregation into classical prey-predator system and show how this prey behavior affects the evolutionary dynamics of the populations.

We aim to understand how prey aggregation alters the outcomes of prey-predator interactions by a minimal model. Let u and v be the densities of prey population and predator population respectively. Without spatial considerations, we assume that the population dynamics are governed by classical Lotka-Volterra system. With the inclusion of spatial motion of populations, which are enclosed in a bounded domain Ω with smooth boundary $\partial\Omega$, we consider a scenario where a spatial flow of prey aggregation occurs, in addition to random diffusions of prey population and predator population. Let $\Omega_\infty=\Omega\times(0,\infty)$. Motivated by [4,5,12,14,23,25] where density-dependent population dispersals are modeled and paper [16] where volume-filling effect is introduced to prevent the blow-up of solutions of a model, we propose the model

$$
\begin{aligned}
&\frac{\partial u}{\partial t}=d_u\Delta u-\nabla\cdot\left(\alpha u\chi(u)\rho(v)\nabla u\right)+ru\left(1-\frac{u}{K}\right)-auv, && (x,t)\in\Omega_\infty,\\
&\frac{\partial v}{\partial t}=d_v\Delta v+buv-cv, && (x,t)\in\Omega_\infty,\\
&\nabla u\cdot\mathbf{n}=\nabla v\cdot\mathbf{n}=0, && x\in\partial\Omega,
\end{aligned}
\tag{1.1}
$$

where d_u and d_v are the random diffusion coefficients of prey and predators respectively, r is the intrinsic growth rate of prey and K is its carrying capacity, a is the attack coefficient of predators, and b is the conversion coefficient of nutrient into recruitment for predators, c is the natural death rate of predators. Furthermore, $\mathbf{n}$ is the outer normal vector of the boundary of domain Ω. It is assumed that all the parameters are positive constants.

The volume-filling coefficient $\chi(u)$ is crucial in practice. Indeed, population aggregation is limited by available space in the area, where each individual occupies a spatial volume. In [16,33], it is assumed that

$$
\chi(u)=\begin{cases}1-\dfrac{u}{u_m}, & \text{if} \quad 0\le u\le u_m,\\ 0, & \text{if} \quad u>u_m,\end{cases}
\tag{1.2}
$$