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Determine Vaccination Priority in a Population of Partially Overlapping Groups

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This paper is dedicated to Professor Zhien Ma on the occasion of his 90th birthday.

Abstract. Personal diverse interests and many group interactions naturally create partially overlapping groups within a population. Those who belong to multiple groups play a crucial role in spreading of infectious diseases across the whole population. We develop an algorithm to decompose the microscopic overlap structure of groups with representing a population of partially overlapping groups as a hypergraph of partially overlapping hyperedges, and characterize it using a newly defined overlap matrix. We formulate a specific multi-group SIR epidemic model, and address a one-time preventive vaccine allocation problem aimed at effectively reducing the basic reproduction number. By leveraging perturbation theory, we derive a principled ranking index to measure the vaccination priority of different groups, and establish a ranking vaccination strategy, which usually outperforms random vaccination strategies as verified by a series of numerical examples. These results offer a theoretical foundation for public health decision-making to develop effective vaccination allocation plans.

AMS subject classifications: 92D30, 34A34

Key words: Vaccination priority, overlap matrix, basic reproduction number, multi-group epidemic model.

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1 Introduction

In social systems of humans, individuals tend to form groups of different sizes, which provides advantages of facilitating communication, enhancing cooperation, and improving adaptability [17]. Individuals within the same group are likely to exhibit similar social behaviours, leading them to engage in group interactions and participate in shared activities. Usually, individuals of a population will join more than one group based on their diverse interests and preferences. For example, a person who joins a basketball club may also join other sports clubs, such as a swimming club, and a mother belonging to a family group may also belong to a friend group. Naturally, the organized groups of a population are often partially overlapping. By definition, groups are partially overlapping if they do not perfectly coincide. In fact, the presence of partially overlapping groups is a common feature of social networks. From the partially overlapping groups of peers, De Giorgi *et al.* [11] demonstrated the peer effects in the choice of college major. More generally in network analysis, detecting overlapping communities (or groups) is a fundamental task with wide-ranging applications, including functional prediction in biology and sub-market identification in economics [12, 15, 31].

Recently, populations composed of partially overlapping groups are often modeled as a hypergraph with partially overlapping hyperedges, where the hyperedges represent the group interactions of the nodes they contain [1, 3, 4, 10]. Especially, Lee *et al.* [19] defined some principled measures and statistically compared the overlaps of hyperedges in real-world hypergraphs. Malizia *et al.* [22] began measuring intra-order hyperedge overlap at both local and global scales, and further identified its role in driving explosive transitions in dynamical processes on hypergraphs. Lamata-Otín *et al.* [18] further incorporated a measure of inter-order hyperedge overlap to capture the downward closure distinguishing simplicial complexes from random hypergraphs, and found that intra- and inter-order overlaps exerted qualitatively distinct effects on the dynamics of coupled chaotic oscillators. However, the impact of such group overlap (indicated by hyperedge overlap) on infectious disease transmission is still poorly understood.

If the vaccine is available for an infectious disease, vaccination serves as an effective means of controlling the epidemic spread. However, vaccine supplies might be limited, particularly for newly identified diseases such as COVID-19 [24]. In this case, it is crucial to identify vaccine prioritization for high-risk individuals or groups and to develop effective allocation strategies. In the context of one-time vaccine allocation, some researchers employed linear or mixed-integer programming models to minimize the quantity of vaccines required or the associated costs, with the goal of reducing the basic reproduction number became below 1 [2, 13, 28]. Rao and Brandeau [25] provided closed-form solutions for the optimal allocation of vaccines, addressing a range of public health objectives, including the reduction of new infections, mortality, years of life lost, and losses in quality-adjusted life years. They also examined the optimal temporal allocation of a limited vaccine supply across population subgroups, balancing distribution between initial doses and multiple booster doses [27]. Numerous studies on COVID-19 developed