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# Mathematical Modeling of Green Sea Turtle Population Dynamics Under Environmental and Thermal Constraints

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This work is dedicated to Professor Zhien Ma, who led me into the realm of mathematical biology, in honor of his 90th birthday.

**Abstract.** Global warming and deteriorating environmental conditions have raised concerns about the persistence of green sea turtle populations, whose reproduction is governed by temperature-dependent sex determination. This study employs mathematical modeling to investigate these ecological challenges. Building on our previous models of green sea turtle population dynamics, we develop a sex-structured and stage-structured life history model that integrates temperature-dependent sex determination and ecological viability, offering a mechanistic framework for understanding green sea turtle population dynamics under climate and environmental stress. Our findings reveal that population dynamics are governed by an Allee-adjusted reproductive number, which accounts for both thermal and environmental influences. Additionally, we conduct a global stability analysis of the collapsed equilibrium using the singular perturbation approach, offering insights into long-term population viability. While additional parameter validation is necessary for definitive conclusions, our results illustrate how climate change and deteriorating environmental conditions shape the long-term viability of green sea turtle populations.

**AMS subject classifications**: 37G10, 92-10 , 92D25, 92D40

**Key words**: Temperature-dependent sex determination, green sea turtle, sex-structured and stage-structured, Allee effects, population collapse, bi-stability.

## 1 Introduction

Green sea turtles (Chelonia mydas) are long-lived marine reptiles, with lifespans exceeding 70 years [5, 15, 26, 32]. As depicted in Fig. 1, their life cycle consists of four distinct stages: egg, hatchling, juvenile, and adult.

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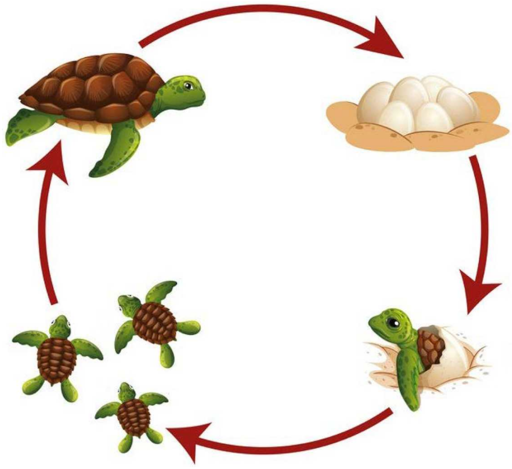

Figure 1: Life cycle of green sea turtles, comprising four stages: egg, hatchling, juvenile, and adult.

They begin as eggs laid on nesting beaches, incubate in the sand, and hatch into small hatchlings that immediately move toward the ocean. As juveniles, they develop in nearshore waters before reaching sexual maturity, often requiring several years or even decades. Adult green turtles, particularly females, make long migrations to nesting beaches to lay eggs, repeating this cycle every few years [32].

The incubation period lasts approximately 45 to 70 days, during which the mean nest temperature determines the sex of the hatchlings, a phenomenon known as temperature-dependent sex determination (TSD). This discovery dates back to Charnier's seminal work [9] in 1966 at the University of Dakar, Senegal. Before then, sex determination in vertebrates was assumed to be strictly genetically controlled. During the past half century, extensive research has expanded this understanding to reptiles.

Comprehensive reviews of TSD include Bull's pioneering work [7], Valenzuela and Lance's field review [38], the introductory book by Lutz *et al.* [24] and Hall's open-access publication [17].

TSD is a process in which environmental temperature influences embryonic development by triggering the production of sex hormones, thus dictating the sex of the embryo. This mechanism, categorized as the determination of environmental sex, is based on physical and biotic factors such as temperature [27, 29]. Every reptile species that exhibits TSD has a thermosensitive period, during which sex differentiation occurs. In turtles, this critical period occurs mid-trimester of the incubation cycle [3].

While many reptile species rely on genetically determined sex via zygotic sex chromosome composition, others – including crocodilians, most turtles, and some lizards – determine sex through incubation temperature during early gonadal differentiation. TSD is prevalent in reptiles and has also been observed in other taxa [10, 23, 31].

During this critical developmental window, slight temperature variations influence enzyme activity and gene expression, leading to differentiation into male or female go-