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AUTOMATED DETECTION AND CHARACTERIZATION OF SINGULARITIES IN FUNCTIONS USING NEURAL NETWORKS-FROM FFT SIGNALS

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Abstract. Singularities, distinctive features signifying abrupt changes in function behavior, hold pivotal importance across numerous scientific disciplines. Accurate detection and characterization of these singularities are essential for understanding complex systems and performing data analysis. In this manuscript, we introduce a novel approach that employs neural networks and machine learning for the automated detection and characterization of singularities based on spectral data obtained through fast Fourier transform (FFT). Our methodology uses neural networks trained on known singular functions, along with the corresponding singularity information, to efficiently identify the location and characterize the nature of singularities within FFT data from arbitrary functions. Several tests have been provided to demonstrate the performance of our approach, including singularity detection for functions with single singularities and multiple singularities.

Key words. Deep neural network, singularity detection, spectral data.

1. Introduction

Singularities, characterized by abrupt changes or discontinuities in functions, are fundamental features encountered in various scientific and engineering domains. Accurate identification and characterization of singularities play a crucial role in understanding the behavior and properties of functions. The potential applications of detecting singularities in Fourier signals are diverse and extend across various fields where signal processing and analysis are essential, such as image processing, biomedical signal processing, environmental monitoring, and financial signal processing. Fourier analysis is widely used in *image processing* for compression, filtering, and feature extraction tasks. Detecting singularities in Fourier-transformed images can help in identifying salient features, discontinuities, or edges in the image domain. These features reinforce object detection, image segmentation, or image enhancement. Fourier analysis is often employed for electroencephalography and electrocardiography signal analysis in *biomedical signal processing.* Detecting singularities in Fourier-based biomedical signals can help recognize abnormal patterns or events, which leads to more accurate diagnosis and effective monitoring of medical conditions. Through Fourier analysis, *environmental monitoring* includes analyzing seismic, oceanographic, or atmospheric data signals. Detecting singularities in such data signals can help predict anomalous events such as earthquakes, tsunamis, or atmospheric disturbances. This identification contributes to early warning systems and disaster management efforts. In addition, Fourier analysis enables time series analysis, volatility modeling, and frequency domain analysis in *finance.* Detecting singularities in financial signal processing can help indicate significant events or anomalies in the data, which can be valuable for risk management, trading strategies, and economic forecasting.

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Despite their importance and wide applications, identifying and characterizing singularities have been challenging, especially in cases with multiple singularities. Fourier analysis suffers from the Gibbs phenomenon, which leads to spurious oscillations around jumps and singularities in discrete Fourier series. These nonphysical oscillations make it challenging to visually identify the characteristics of the jumps and singularities. Various data reconstruction techniques, such as Gegenbauer reconstructions, have been proposed to recover spectral accuracy up to the jumps [28, 24, 27, 25, 26, 1, 30] and singularities [11, 12, 35]. These techniques have found widespread applications in post-processing numerical simulations [42, 23, 29, 34] and image reconstructions [2, 3, 4, 8]. However, such reconstruction techniques still require accurate information on the location and type of trouble points. Various edge detection methods have been devised for Gegenbauer reconstructions on discontinuous functions [21, 22, 18] based on truncated spectral expansions or collocation point values. These methods have proved successful across diverse input data types, including spectral partial sums and discontinuous Galerkin solutions [8, 7, 5, 45, 6, 19, 43, 41, 46, 14, 39, 20, 16, 17]. However, these methods focus on detecting discontinuities (not general singularities). A local singularity detection algorithm [37] has been developed to obtain high-order evaluations of singularity characteristics, such as location and exponent, through locally supported quasi-interpolation of univariate nonsmooth functions. Moreover, wavelet transforms have been employed effectively to identify Lipschitz regularity and characterize singularities in irregular signals, which leads to successful applications in signal denoising [38, 9, 44]. These techniques have been applied to various domains, including sinogram imaging [33], seismic imaging [32], and cone beam CT breast imaging [49]. Nevertheless, identifying various types of singularities and detecting multiple singularities in a function still need to be studied.

Neural network techniques have shown remarkable success in many fields, including computer vision [31], pattern recognition [40], natural language processing [15], and other tasks related to artificial intelligence. With the success of feature extraction in various research areas [13], neural network techniques have attracted significant attention for data-related applications. Machine learning techniques have recently been used to detect singularities in data, e.g., topological data and patterns [36, 47, 48]. Therefore, this paper proposes a neural network-based approach to automating the detection and characterization of singularities in functions. Our methodology uses a training dataset of randomly generated functions with known singularity locations and exponents. Fourier coefficients are computed for these functions, capturing their frequency domain characteristics. These coefficients, along with the corresponding singularity information, are used to train a neural network model detecting the underlying patterns and relationships. The neural network architecture is designed to handle the detection of one or multiple singularities, taking the Fourier coefficients as inputs and predicting both the singularity locations and exponents. Using neural network structures, we present three singularity detection models:

(1) *Single singularity detector*: This detector approximates the singularity locations and exponents for functions with single singularities.
(2) *Multiple singularities detector*: This detector extends the above detector to capture the locations and exponents of multiple singularities in singular functions.
(3) *Multiple singularities detector with splitting strategy*: This detector enhances the above detector by employing the splitting strategy in detecting