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Global Convergence of a New Conjugate Gradient Method with Wolfe Type Line Search+

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Abstract. In this paper, a new nonlinear conjugate gradient methods with wolfe type line search for solving unstrained optimization problems is proposed. The direction generated by the new methods produce sufficient descent search direction. Under some conditions, we give the global convergence results for the new nonlinear conjugate gradient methods with Wolfe type line search. Finally, the numerical results show that the new method is also efficient for general unconstrained optimizations.

Key Words: Unconstrained optimization; nonlinear conjugate gradient method; line search; global convergence

1. Introduction

In this paper, we consider the following large scale unconstrained optimization problem

$$\min_{x\in R^n} f(x) \tag{1.1}$$

where $f: R^n \to R$ is continuously differentiable and its gradient $g(x)=\nabla f(x)$ is available. This problem has been well studied. Iterative methods are widely used for solving (1.1) and the iterative formula is given by

$$x_{k+1} = x_k + \alpha_k d_k \tag{1.2}$$

where $x_k \in R^n$ is the k-th approximation to the solution of (1.1), α_k is a stepsize obtained by carrying out a line search and d_k is a search direction.

Due to the simplicity of its iteration and low memory requirements, the nonlinear conjugate gradient method are one of the most famous methods for solving the above unconstrained optimization problem (1.1), especially in case of the dimension n of $f(x)$ is large, which are often proposed in scientific and engineering computation. The search direction d_k is defined by

$$d_k = \begin{cases} -g_k, & k=1, \\ -g_k + \beta_k d_{k-1}, & k \ge 2. \end{cases} \tag{1.3}$$

where β_k is a scalar and $g_k = g(x_k)$ .Different conjugate gradient methods correspond to different scalar β_k in (1.3), i.e. [1-6]

$$\beta_k^{FR} = \frac{g_k^T g_k}{g_{k-1}^T g_{k-1}}, \text{ (Fletcher-Reeves)}, \quad \beta_k^{HS} = \frac{g_k^T (g_k - g_{k-1})}{(g_k - g_{k-1})^T d_{k-1}}, \text{ (Hestenes-Stiefel)},$$

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$$\beta_k^{PRP} = \frac{g_k^T(g_k - g_{k-1})}{g_{k-1}^T g_{k-1}}, \text{ (Polak-Ribiere-Polyak)}, \ \beta_k^{CD} = -\frac{g_k^T g_k}{d_{k-1}^T g_{k-1}}, \text{ (Conjugate Descent)},$$

$$\beta_k^{LS} = -\frac{g_k^T(g_k - g_{k-1})}{d_{k-1}^T g_{k-1}}, \text{ (Liu-Storey)}, \ \beta_k^{DY} = \frac{g_k^T g_k}{(g_k - g_{k-1})^T d_{k-1}}, \text{ (Dai-Yuan)}.$$

The nonlinear conjugate gradient methods and the global convergence results about Fletcher-Reeves(FR) method, Polak-Ribiere-Polyak(PRP) method, Hestenes-Stiefel(HS)method, Dai-Yuan(DY) method, Conjugate Descent(CD) method and Liu-Storey(LS) method can see [5-11]. Quite recently, a new nonlinear conjugate gradient method was proposed in [12]. The search direction β_k are given by the following way

$$\beta_k = \frac{1}{d_{k-1}^T y_{k-1}}(y_{k-1} - 2d_{k-1}\frac{\|y_{k-1}\|^2}{d_{k-1}^T y_{k-1}})^T g_k, \tag{1.4}$$

where $y_{k-1} = g_k - g_{k-1}$, $\|\cdot\|$ stands for the Euclidean norm.

In this paper, based on the above new nonlinear conjugate gradient method in [12], under some mild conditions, we give the global convergence of the new nonlinear conjugate gradient method with Wolfe type line search. The rest of the paper is organized as follows. In Section 2, we present the Wolfe type line search and the new nonlinear conjugate gradient method. In Section 3, we present the global convergence results of the new nonlinear conjugate gradient method with Wolfe type line search. Numerical results and some discussions are given in the Section 4.

2. New nonlinear conjugate gradient method

In this section, we will give the following assumptions on objective function $f(x)$ in (1.1). The two assumptions have been often used in the literature to prove the global convergence of nonlinear conjugate gradient methods with exact and inexact line searches for unconstrained optimization problems.

Assumption 2.1. (i) The level set $L_0 = \{x \in R^n \mid f(x) \le f(x_0)\}$ is bounded. (ii) In some neighborhood U of L_0, $f(x)$ is continuously differentiable and its gradient is Lipschitz continuous, namely, there exists a constant $L > 0$ such that

$$\|g(x) - g(y)\| \le L\|x - y\|, \quad \forall x, y \in U.$$

And $g(x)$ also satisfies the following condition

$$(g(x) - g(y))(x - y) \ge \tau\|x - y\|^2,$$

for $\tau > 0$ and $\forall x, y \in U$.

Now we consider the modified new conjugate gradient method for (1.1) with Wolfe type line search. Firstly, we give the following Wolfe type line search (we have proposed in [14]).

The line search is to choose $\alpha_k > 0$ such that

$$f(x_k + \alpha_k d_k) - f(x_k) \le -\rho\alpha_k^2\|d_k\|^2, \tag{2.1}$$

$$g(x_k + \alpha_k d_k)^T d_k \ge -2\sigma\alpha_k\|d_k\|^2, \tag{2.2}$$

where $0 < \rho < \sigma < 1$.

Now we present the new conjugate gradient methods as follows.

Algorithm 2.1.

Step 0. Given $x_0 \in R^n$, set $d_0 = -g_0$. If $g_0 = 0$, then stop.

Step 1. Find $\alpha_k > 0$ satisfying the Wolfe type line search (2.1), (2.2), by (1.2), x_{k+1} is given. If $g_{k+1} = 0$, then stop.