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A Real Coded Genetic Algorithm to Entropy Bimatrix Game: Fuzzy Programming Technique

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Abstract. Here, we propose a mathematical model to analyze bimatrix game under entropy environment. In this new approach, the entropy function for each player are considered as objectives to the bimatrix game. This formulated model is named as Entropy Bimatrix Game Model which is a multi-objective non linear problem. To solve this kind of model, we have introduced a new solution technique, which determines the feasibility of fuzzy multiobjective non linear programming via a revised Genetic Algorithm (GA). By a real coded Genetic Algorithm, we obtained the bounds of objectives of said model and then applied fuzzy programming to determine the Nash equilibrium solution. To illustrate the methodology, numerical examples are included.

Keywords: Bimatrix Game, Nash Equilibrium, Entropy, Fuzzy Programming, GA.

1. Introduction

Every probability distribution has some "uncertainty" associated with it. The concept of "entropy" is introduced to provide a quantitative measure of uncertainty. Entropy models are emerging as valuable tools in the study of various social and engineering problems. The maximum entropy principle initiated by Jaynes'[8] is a powerful optimization technique of determining the distribution of random system in the case of partial or incomplete information or data available in the system. The principle has now been broadened and extended and has found wide applications in different fields of science and technology.

Two-person zero-sum game models are accurate when stakes are small monetary amounts. But in reality sense, when the stakes are more complicated, as often in economic situations, it is not generally true that the interests of the two players are exactly opposed. Such type of game models are non-cooperative game model. In other words, such situations give rise to two-person non-zero sum game, called bimatrix games. A bimatrix game can be considered as a natural extension of the matrix game, to cover situations in which the outcome of a decision process does not necessarily dictate the verdict that what one player gains the other one has to lose.

In bimatrix game, we see that family of probability distributions of strategies of every player are consistent with given information, we choose the distribution whose uncertainty or entropy is maximum. Each player is interested in making moves which will be as surprising and as uncertain to the other player as possible. For this reason, the players are involved in maximizing their entropies. Consequently, in the mathematical models of bimatrix game are incorporated an entropy function as one of their objectives. These models are known as entropy bimatrix game model.

In conventional mathematical programming, the coefficients or parameters of the bimatrix game models are assumed to be deterministic and fixed. But, there are many situations where they may not be exactly known i.e., they may have some uncertainty in nature. Thus the decision-making methods under uncertainty are needed. The fuzzy programming has been proposed from this viewpoint. In fuzzy programming problems, the coefficients, constraints and the goals are viewed as fuzzy number or fuzzy set. In decision-making process, first Bellman and Zadeh [20] introduced fuzzy set theory. Tanaka applied the concepts of fuzzy sets to decision making problems by considering as fuzzy goals[18] and Zimmermann[21] showed that the classical algorithms could be used to solve multi-objective fuzzy linear programming problems.

In this paper, some references are presented including their work. Borm, Vermeulen and Voorneveld[2] analyzed the structure of the set of equilibria for the two-person multicriteria game. It turns out that the classical result for the set of equilibria for bimatrix games is valid for multicriteria games if one of players has two pure strategies. In another paper[19] they generalised some axioms of the Nash equilibria and it was shown that there exists no consistent refinement of Nash equilibria concept that satisfy individual rationality and non emptiness on a reasonably large class of games (Borm, Vermeulen and Voorneveld 2003). Nishizaki and Sakawa ([11],[12],[13]) proposed the resolution approach which can be regarded as a paradigm for bimatrix multi-objective non-cooperative game. Roy[17] presented the study of two different solution procedures for the two-person bimatrix game. The first solution procedure is applied to the game on getting the probability to achieve some specified goals along the player's strategy. The second specified goals along with the player's strategy by defining the fuzzy membership function to the pay-off matrix of the bimatrix game. Das and Roy ([5],[15]) have presented some two-persons zero sum game under entropy environment.

Several methodologies have been proposed to solve bimatrix game. Most of these methods are based on the concept of Pareto-optimal security strategies for linear models. However, no studies have been made on bimatrix entropy game. Genetic Algorithm(GA) and Fuzzy programming technique play an important role for determining the corresponding solution of the proposed model.

2. Mathematical Model of a Bimatrix Game

A bimatrix game can be considered as a natural extension of the matrix game. A two-person non zero-sum game can be expressed by a bimatrix game, comprised of two $m \times n$ dimensional matrices, namely A and B, where

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

$$B = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{m1} & b_{m2} & \cdots & b_{mn} \end{bmatrix}$$

If player PI adopts the strategy "row" and player PII adopts the strategy "column" then denotes the expected payoff for player PI and denotes the expected payoff for player PII .

Definition 1:(Mixed Strategies)

The mixed strategies of the bimatrix game for player PI and PII are defined as follows:

$$Y = \{y \in R^m; \textstyle\sum_{i=1}^{m} y_i = 1; y_i \geq 0, i = 1,2,\ldots,m\} \quad (1)$$

$$Z = \{z \in R^n; \textstyle\sum_{j=1}^{n} z_j = 1; z_j \geq 0, j = 1,2,\ldots,n\} \quad (2)$$

Definition 2:(Nash equilibrium Solution)

For bimatrix game of two players, the Nash equlibrium solution (y^*, z^*) is found, if

$$y^{*t}Az^* \geq y^tAz^* \quad (3)$$

$$y^{*t}Bz^* \geq y^{*t}Bz \quad (4)$$

where $y \in Y$ and $z \in Z$; t denotes the transepose of a matrix.

Definition 3:(Expected Payoffs of Players)

If the mixed strategies are proposed by player PI and PII, then the expected payoff of player PI is y^tAz^*; the expected payoff of player PII is $y^{*t}Bz$. Therefore, the two person bimatrix game with mixed strategies can be formulated as follows:

$$\max_{y \in Y} y^tAz^* \quad (5)$$

and