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Some Explicit Class of Hybrid Methods for Optimal Control of Volterra Integral Equations

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Abstract. In this paper, we first extend and analyze the steepest descent method for solving optimal control problem for systems governed by Volterra integral equations. Then, we present some hybrid methods based on the extended steepest descent and two-step Newton methods, to solve the problem. The global convergence results are also established using some mild assumptions and conditions. Numerical results show that the proposed hybrid methods are more powerful and faster than the extended steepest descent method.

Keywords: Optimal control, Steepest descent, Two-step Newton's method, Volterra integral equation.

1. Introduction

The classical theory of optimal control was originally developed to deal with systems of controlled ordinary differential equations [9]. A wide class of control systems can be described by Volterra integral equations instead of ordinary differential equations. It is well-known that Volterra integral equations can be used to model many classes of phenomena, such as population dynamics, continuum mechanics of materials with memory economic problems, the spread of epidemics, non-local problems of diffusion and heat conduction problem. An excellent survey on applications of Volterra integral equation can be found in [6] and [8] .

The problem of optimal control of systems governed by Volterra integral equations has been studied by many authors e.g. Vinokurov [10], Medhin [11], Schmidt [13] and Belbas [3, 4, 5]. The methods that are usually employed for solving this problem are based on the necessary conditions obtained using Pontryagin's maximum principle. Belbas in [3] presented a method based on discretization of the original Volterra integral equation and a novel type of dynamic programming in which the Hamilton-Jacobi function is parametrized by control function. In more recent work of Belbas [4], the controlled Volterra integral equations are approximated by a sequence of controlled ordinary differential equations and the resulting approximating problems can then be solved by dynamics programming methods for ODEs controlled systems. The interested reader may found some references on optimal control of Volterra integral equations by methods other than dynamic programming in [5,7, 10, 11].

Due to the difficulties in obtaining analytical solution for the problem, the numerical methods have been usually interested. Belbas in [5] described several iterative schemes for solving Volterra optimal problems and analyzed the conditions that guarantee the convergence of the methods. Schmidt in [14], proposed some direct and indirect numerical methods for solving optimal control problems governed by ODEs as well as integral equations.

In this paper, we are going to provide some explicit iterative methods based on the necessary conditions for solving Volterra optimal control problems in which the control variables are not constrained by any boundaries. We first generalized the Steepest Descent (**SD)** method for solving the problem and then hybridize the SD and Two-Step Newton (**TSN)** methods in order to efficiently solving the Volterra optimal control problem. The proposed hybrid method integrates the SD and TSN methods to obtain global convergence results together with fast convergence rate. Our numerical results show that the hybrid schemes are more powerful and faster than the SD method.

The paper is organized as follows: the Volterra optimal control problem and some elementary related results are stated in section 2. Section 3 is devoted to describe and analyze the generalized SD and TSN methods. The new hybrid schemes based on SD and TSN methods are also constructed in this section. The global convergence of the proposed hybrid methods is analyzed in section 4 and finally some numerical results are given in section 5 to show the efficiency of the proposed hybrid methods in comparison with the SD method.

2. Problem Statement

Consider a controlled Volterra integral equation of the form

$$x(t) = x(a) + \int_a^t f\big(t,s,x(s),u(s)\big)\,ds, \tag{1}$$

where the continuous real-valued functions $x(t)$ and $u(t)$ are the state of the controlled system and the control function, respectively. It is assumed that the state and control variables are not constrained by any boundaries. In this paper, we consider the optimal control problem in which the cost functional J defined by

$$J = \int_a^b F\big(t,x(t),u(t)\big)dt, \tag{2}$$

is minimized under the Volterra integral equation given by (1). We are looking for the vector (x^*,u^*) which solves the following problem

$$\begin{aligned} &max \qquad J_1 = -J, \\ &s.t. \qquad x(t) = x(a) + \int_a^t f\big(t,s,x(s),u(s)\big)ds. \end{aligned} \tag{3}$$

It is well known that the Pontryagin maximum principle, or simply maximum principle, gives the necessary conditions for the optimal vector (x^*,u^*) of the problem (3), so through out the paper we assumed that the following conditions are satisfied:

$\boldsymbol{H_1}$: The conditions that guarantee the existence of a unique continuous solution to the integral equation (4), which include continuity of the function f for all s,t with $s \le t$, together with its Lipschitz condition.

$\boldsymbol{H_2}$: The partial derivatives f_x and f_u exist and are continuous, and for all t,s with $t \le s$, $f(t,s,x,u) = 0$.

$\boldsymbol{H_3}$: $F(t,x,u)$ is a smooth function.

Following [15], using these assumptions, the Pontryagin maximum principle can be stated as follows : Suppose that the function $u^*(t)$, $a \le t \le b$, and related state function $x^*(t)$, solve the problem (3). Then, there exist a continuous multiplier function $\lambda^*(t)$, and a Hamiltonian $H(t,x,u,\lambda)$, defined by