

Synchronization of different 3D chaotic systems by generalized active control

Mohammad Ali Khan

Department of Mathematics, Garhbeta Ramsundar Vidyabhaban,

Garhbeta, Paschim Medinipur, West Bengal, India

(Received May 19, 2012, accepted September 13, 2012)

Abstract. This paper designs a scheme for controlling a chaotic system to a period system using active control technique. We discuss this taking chaotic Genesio system as example. We have also discussed the synchronization scheme between two different coupled chaotic systems (Genesio and Nuclear spin generator(NSG)system) as well as two identical coupled chaotic systems (Four-scroll attractor) via active control. Numerical Simulation results are presented to show the effectiveness of the proposed scheme.

Keywords: Chaotic system, Chaos control, Chaos synchronization, Active control, Genesio system, NSG system, Four-scroll attractor.

1. Introduction

Chaos synchronization is an important topic in the non-linear science. In 1990, Pecorra and Carroll [1] introduced the idea of chaos synchronization. Two or more coupled chaotic systems are called synchronized if their behaviors are closely related. Chaos synchronization has received much attention due to its applications in many area such secure communication, information processing, biological systems and chemical reactions. Usually two dynamical systems are called synchronized if the distance between their corresponding states converges to zero as time goes to infinity. This type of synchronization is called identical synchronization. A generalization, of this concept for unidirectionally coupled dynamical systems was proposed by Rulkov et.al.(1995) [2] where two coupled systems are called synchronized if a static functional relationship exists between the states of the systems. This kind of synchronization is called generalized synchronization(GS). In 2009 synchronization in unidirectionally coupled Rossler system was proposed by Khan and Mandal [3] and also synchronization in bidirectionally coupled systems reported by Tarai et.al. [4,5]. In 2011 Khan et.al. [6] have discussed various type of control for controlling chaos in a unified chaotic system and recently in 2012 generalized anti-synchronization of different chaotic systems was proposed by Khan et.al. [7]. In 1996, Kocarev and Parlitz [8] formulated a condition for the occurrence of GS for the master and slave system. Using technique from active control theory, ER-Weibai and Karl.E.Lonngren(1997) [9] demonstrate that a coupled Lorenz systems can synchronize. Further ER-Weibai and Karl.E.Lonngren [10] have investigated sequential synchronization of two Lorenz system using active control. In 2001, synchronization of Rossler and Chen chaotical dynamical syatems using active control was studied by Agiza and Yassen [11]. In 2002 Ming-Chuang Ho, Yao-Chen Hung [12] generalized the technique of active control theory and applied them to synchronize two different systems. Sinha et.al. [13] proposed a general approach in the design of active controllers for non-linear systems exhibiting chaos in 2003. Synchronization of two chaotic four-dimensional systems using active control technique was proposed by Youming and Wenxian in 2007 [14]. In 2009 Sudheer and Sabir [15] was investigated hybrid synchronization of hyperchaotic Lu system via active control. Recently in 2012 Shahzad [16] was investigated chaos synchronization of an ellipsoidal satellite via active control. Synchronization scheme for many coupled chaotic systems are not explored till now. Synchronization strategy for coupled chaotic Genesio system is very important from the theoretical point of view and this strategy is reported by us in this

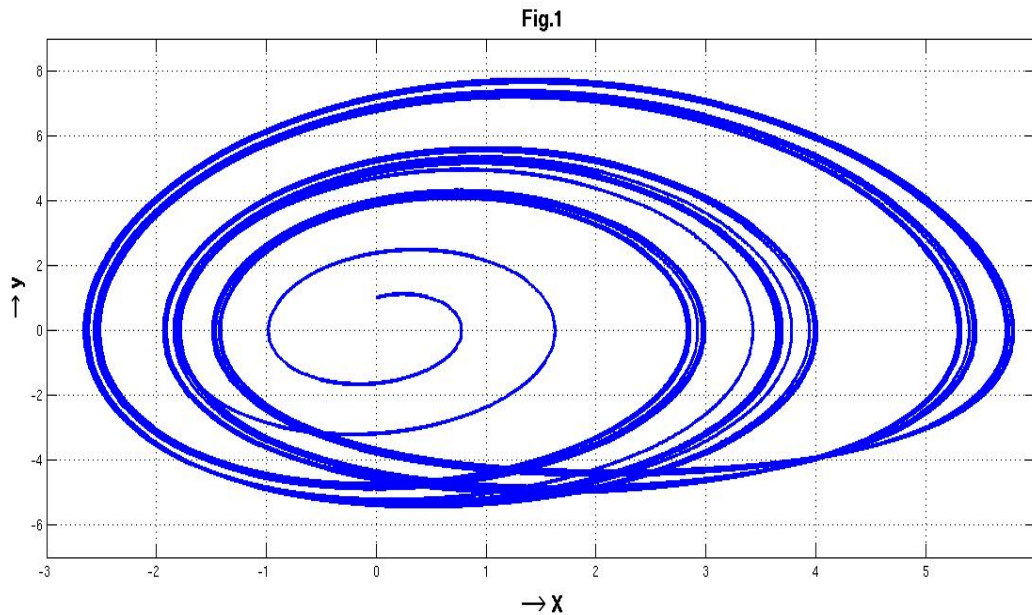
paper. The synchronization strategy between two identical coupled chaotic systems as well as two different chaotic systems are also very important from the application point of view. This motivates us to study the synchronization between Genesio system and Nuclear spin generator systems and synchronization between Four-scroll attractor systems in this paper.

2. Genesio system

The Genesio system, proposed by Genesio et.al [17], it captures many features of chaotic systems. It includes a simple square part and three simple ordinary differential equations that depend on three positive real parameters. The dynamical equations are described by

$$\begin{aligned}\dot{x} &= y \\ \dot{y} &= z \\ \dot{z} &= -cx - by - az + x^2\end{aligned}\quad (1)$$

where x , y and z are state variables and a , b and c are positive constants, satisfying $ab < c$. The Genesio system exhibits a chaotic attractor at the parameter values $a=1.2$, $b=2.92$ and $c=6$ shown in Fig.1.



2.1. Control of chaotic Genesio system to a period system

In order to observe the control behavior we take three dimensional period systems as master system with three state variables denoted by the subscript 1 and the slave systems denoted by the subscript 2. The initial conditions for the master system are different from that of the slave system. The master and slave systems are defined as follows

Master system

$$\begin{aligned}\dot{x}_1 &= y_1 \\ \dot{y}_1 &= z_1 \\ \dot{z}_1 &= -y_1\end{aligned}\quad (2)$$

Slave system

$$\begin{aligned}\dot{x}_2 &= y_2 + u_a \\ \dot{y}_2 &= z_2 + u_b \\ \dot{z}_2 &= -cx_2 - by_2 - az_2 + x_2^2 + u_c\end{aligned}\quad (3)$$

where $U = [u_a, u_b, u_c]^T$ is the controller function introduced in the slave system. Using active control techniques, we subtract equation (2) from equation (3) and get