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A Spline based computational simulations for solving self-adjoint singularly perturbed two-point boundary value problems

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Abstract. In this paper, we proposed a spline based computational simulations for solving self-adjoint singularly perturbed two-point boundary value problems. The original problem is reduced to its normal form and the reduced boundary value problem is treated by using difference approximations via cubic splines in tension. The convergence of the method is analyzed. Some numerical examples are given to demonstrate the computational efficiency of the present method.

Keywords: Self-adjoint, Singularly Perturbed Problems, Difference Approximations, Cubic Spline in Tension.

1. Introduction

We consider the following self-adjoint singularly perturbed two-point boundary value problem:

$$Ly \equiv -\varepsilon(a(x)y')' + b(x)y = f(x) \text{ on } [0, 1] \text{ with } y(0) = \alpha,\ y(1) = \beta, \tag{1.1}$$

where α, β are given constants and ε is a small positive parameter. We also assume that the coefficients *a(x), b(x)* are sufficiently smooth function satisfying

$$a(x) \geq \xi_0 > 1,\ a(x) \geq 0, b(x) \geq \xi_1 > 0 \tag{1.2}$$

where ξ_0 and ξ_1 are some positive constants. Under these conditions operator L admits a maximum principle [1]. These type of problems arise frequently in fluid mechanics, aerodynamics, plasma dynamics, magneto hydrodynamics, oceanography, optimal control, chemical reactions, etc., In recent years, seeking numerical solutions of singularly perturbed boundary value problems has been the focus of a number of authors. Niijima [2, 3] produced a uniformly second order accurate difference schemes where as Miller [4] gave sufficient conditions for the uniform first order convergence of a general three-point difference schemes. Boglayev [5] discussed a variational difference scheme for solving boundary value problems with a small parameter in the highest derivative. Schatz and Wahlbin [6] used finite element techniques for solving singularly perturbed reaction diffusion problems in two and one dimension. In [7] a method based on spline collocation was presented for solving singularly perturbed boundary value problems. Cubic spline in compression for second order singularly perturbed boundary value problems was presented in [8]. Kadalbajoo and Devendra Kumar [9] presented a numerical method based on finite difference method with variable mesh for solving second order singular perturbed self-adjoint two-point boundary value problems. In [10] a fitted operator finite difference method via the standard Numerov's method was presented for solving self-adjoint singular perturbation problems. Riordan and Stynes [11] presented a uniformly accurate finite element method for solving singularly perturbed one dimensional reaction-diffusion problem. In [12], a spline approximation method was presented for solving self-adjoint singular perturbation problem on non-uniform grids. Lubuma and Patidar [13] presented uniformly convergent non-standard finite difference methods for solving self-adjoint singular perturbation problems. Mishra et.al [14] extended the initial value

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technique to self-adjoint singularly perturbed boundary value problems. Recently, variable mesh finite difference method [15] was extended for solving self-adjoint singularly perturbed two-point boundary value problems.

In order to solve the self-adjoint singularly perturbed problem, first we reduce equation (1.1)-(1.2) to its normal form and then the reduced problem is treated by spline method. In general finding numerical solution of a second order boundary value problem with y' term is more difficult as compared to a second order boundary value problem with absence of y' term. Therefore, it is better to convert the second order boundary value problem with y' term to the second order boundary value problem without y' term i.e to its normal form. In this paper, we have presented computational simulations of self-adjoint singular perturbed two-point boundary value problems via spline method. Convergence of the method is analyzed and some numerical evidences are included to show the applicability and efficiency of the method..

2. Description of the Method

We consider the following self-adjoint singularly perturbed two-point boundary value problem:

$$-\varepsilon(a(x)y')' + b(x)y = f(x) \text{ on } [0, 1] \text{ with } y(0) = \alpha,\ y(1) = \beta, \tag{2.1}$$

where α, β are given constants and ε is a small positive parameter. We also assume that the coefficients *a(x), b(x)* are sufficiently smooth function satisfying

$$a(x) \geq \xi_0 > 1, a(x) \geq 0, b(x) \geq \xi_1 > 0 \tag{2.2}$$

where ξ_0 and ξ_1 are some positive constants. Equation (2.1) can be written as

$$-\varepsilon a(x)y''(x) - \varepsilon a'(x)y'(x) + b(x)y(x) = f(x) \qquad \text{or}$$

$$y''(x) + \frac{a'(x)}{a(x)}y'(x) - \frac{b(x)}{\varepsilon a(x)}y(x) = -\frac{f(x)}{\varepsilon a(x)} \qquad \text{or}$$

$$y''(x) + p(x)y'(x) + q(x)y(x) = r(x) \tag{2.3}$$

where $p(x) = \dfrac{a'(x)}{a(x)},\ q(x) = -\dfrac{b(x)}{\varepsilon a(x)}$ *and* $r(x) = -\dfrac{f(x)}{\varepsilon a(x)}$

Consider the transformation

$$y(x) = U(x)V(x) \tag{2.4}$$

Then equation (2.3) can be written as its normal form as

$$V''(x) + A(x)V(x) = G(x) \tag{2.5}$$

$$\text{with } V(0) = \frac{y(0)}{U(0)} = \gamma_0,\ \ V(1) = \frac{y(1)}{U(1)} = \gamma_1, \qquad \gamma_0, \gamma_1 \in R \tag{2.6}$$

where $A(x) = q(x) - \frac{1}{2}p'(x) - \frac{1}{4}(p(x))^2$