ISSN 1746-7659, England, UK
Journal of Information and Computing Science
Vol. 8, No. 2, 2013, *pp*. 083-089

Rational solitary wave solutions for some nonlinear differential difference equations

Khaled A. Gepreel[1, 2]

[1] Mathematics Department, Faculty of Science, Zagazig University, Egypt.

[2]Mathematics Department, Faculty of Science, Taif University, Saudi Arabia.

(*Received May 4, 2012, accepted January 3, 2013*)

Abstract. In this article, we put a direct method to construct the rational solitary wave solutions for some nonlinear differential difference equations in mathematical physics which may be called the rational solitary wave difference method. We use the proposed method to construct the rational solitary exact solutions for some nonlinear differential difference equations via the lattice equation, the discrete nonlinear Klein Gordon equation. The proposed method is more effective and powerful to obtain many rational solitary exact solutions for nonlinear differential difference equations.

Keywords: Solitary wave solutions, Traveling wave solutions, The lattice equation, The discrete Klein Gordon equation.

1. Introduction

It is well known that the investigation of differential difference equations (DDEs) which describe many important phenomena and dynamical processes in many different fields, such as particle vibrations in lattices, currents in electrical networks, pulses in biological chains a many others and so on, has played an important role in the study of modern physics. Unlike difference equations which are fully discredited, DDEs are semi- discredited with some (or all) of their special variables discredited while time is usually kept continuous. DDEs also play an important role in numerical simulations of nonlinear partial differential equations (NLPDEs), queuing problems, and discretization in solid state and quantum physics.

Since the work of Fermi, Pasta, and Ulam in the 1960s [1], DDEs have been the focus of many nonlinear studies. On the other hand, a considerable number of well-known analytic methods are successfully extended to nonlinear DDEs by researchers [2–17]. However, no method obeys the strength and the flexibility for finding all solutions to all types of nonlinear DDEs. Zhang etal. [18] and Aslan [19] used the (G'/G)-expansion method to some physically important nonlinear DDEs. Qiong etal. [12] constructed the Jacobi elliptic solutions for nonlinear DDEs. Recently Zhang etal [20] and Gepreel [29,30] have used the Jacobi elliptic function method for constructing new and more general Jacobi elliptic function solutions of some nonlinear difference differential equations. The main objective of this paper, is to modify the rational solitary wave method which discussed by Xie [31] to solve the nonlinear differential difference equations instead of solving the nonlinear partial differential equations which my be called rational solitary wave difference method. We use the proposed method to calculate the rational solitary wave solutions for some nonlinear DDEs in mathematical physics via the lattice equation and the discrete nonlinear Klein Gordon equation.

2. Description of the rational solitary wave difference method

In this section, we would like to outline on algorithm for using the rational solitary wave difference method to solve nonlinear DDEs. For a given nonlinear DDEs

$$\begin{aligned}&\Delta(u_{n+p_1}(x),\dots,u_{n+p_k}(x),u'_{n+p_1}(x),\dots,u'_{n+p_k}(x),\dots,u^{(r)}_{n+p_1}(x),\dots,u^{(r)}_{n+p_k}(x),\\&v_{n+p_1}(x),\dots,v_{n+p_k}(x),v'_{n+p_1}(x),\dots,v'_{n+p_k}(x),\dots,v^{(r)}_{n+p_1}(x),\dots,v^{(r)}_{n+p_k}(x),\dots)=0,\end{aligned} \tag{1}$$

where $\Delta=(\Delta_1,...,\Delta_g), x=(x_1,x_2,...,x_m),\ n=(n_1,...,n_Q)$ and $g,m,Q,p_1,...,p_k$ are integers, $u_i^{(r)},v_i^{(r)}$ denotes the set of all r th order derivatives of u_i,v_i with respect x.

The main steps of the algorithm for the rational solitary wave difference method to solve nonlinear DDEs are outlined as follows:

Step 1. We take the traveling wave solutions of the following form:

$$u_n(x)=U(\xi_n),\quad v_n(x)=V(\xi_n),\ ...\quad , \tag{2}$$

where

$$\xi_n=\sum_{i=1}^{Q}d_i n_i-\sum_{j=1}^{m}c_j x_j+\xi_0, \tag{3}$$

and $d_i(i=1,..,Q)$, $c_j,(j=1,...,m)$, the phase ξ_0 are constants to be determined later. The transformations (2) is reduced Eqs.(1) to the following nonlinear differential difference equations

$$\begin{aligned}&\Omega(U(\xi_{n+p_1}),...,U(\xi_{n+p_k}),U'(\xi_{n+p_1}),...,U'(\xi_{n+p_k}),...,U^{(r)}(\xi_{n+p_1}),...,U^{(r)}(\xi_{n+p_k}),\\&V(\xi_{n+p_1}),...,V(\xi_{n+p_k}),V'(\xi_{n+p_1}),...,V'(\xi_{n+p_k}),...,V_{n+p_1}^{(r)}(\xi_{n+p_1}),...,V_{n+p_k}^{(r)}(\xi_{n+p_k}),...)=0,\end{aligned} \tag{4}$$

where $\Omega=(\Omega_1,...,\Omega_g)$.

Step 2. We suppose the rational solitary wave series expansion solutions of Eqs (4) in the following form:

$$\begin{aligned}U(\xi_n)&=\sum_{i=0}^{N}a_i[g(\xi_n)]^i+\sum_{j=1}^{N}b_j[g(\xi_n)]^{j-1}f(\xi_n),\\V(\xi_n)&=\sum_{i=0}^{L}\alpha_i[g(\xi_n)]^i+\sum_{j=1}^{L}\beta_j[g(\xi_n)]^{j-1}f(\xi_n),....,\end{aligned} \tag{5}$$

with

$$f(\xi_n)=\frac{1}{A\tanh(\xi_n)+B\sec h(\xi_n)},\quad g(\xi_n)=\frac{\sec h(\xi_n)}{A\tanh(\xi_n)+B\sec h(\xi_n)}, \tag{6}$$

which satisfy

$$\begin{aligned}&f'(\xi_n)=-Ag^2(\xi_n)+\frac{Bg(\xi_n)}{A}[1-Bg(\xi_n)],\quad g'(\xi_n)=-Af(\xi_n)g(\xi_n),\\&f^2(\xi_n)=g^2(\xi_n)+\frac{1}{A^2}[1-Bg(\xi_n)]^2,\\&f(\xi_n\pm d)=\frac{A^2f(d)f(\xi_n)\pm[1-Bg(\xi_n)][1-Bg(d)]}{A^2f(d)[1-Bg(\xi_n)]\pm A^2f(\xi_n)[1-Bg(d)]+BA^2g(\xi_n)g(d)},\\&g(\xi_n\pm d)=\frac{g(\xi_n)g(d)}{f(d)[1-Bg(\xi_n)]\pm f(\xi_n)[1-Bg(d)]+Bg(\xi_n)g(d)},\end{aligned} \tag{7}$$

where $a_i,\alpha_i,\ b_j,\beta_j,A,B$ are constants to be determined.

Also, we can assume that

$$f(\xi_n)=\frac{1}{A\tan(\xi_n)+B\sec(\xi_n)},\quad g(\xi_n)=\frac{\sec(\xi_n)}{A\tan(\xi_n)+B\sec(\xi_n)} \tag{8}$$