A Quasi-Radial Basis Function Method for European Option Pricing

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Abstract: In this paper, we propose a meshless method for option pricing which uses the radial basis quasi-interpolation method to solve the Black-Scholes equation. The quasi-interpolation operator is used to force the first and second derivatives of stock prices in spatial direction and the forward difference method is used in time direction. Its convergence of order $O\left(\Delta t + h^{\frac{2}{3}}\right)$ in l_{∞} −norm is also derived in the paper. The advantage of this method is that it can fit the scattered data well, which makes it be a good approximation method for the option prices that fluctuate randomly. The feasibility of the proposed method is verified by numerical examples of uniform points and scattered points. The results show that this method has a good fitting effect on option prices.

Keywords: Quasi-interpolation; Meshless method; Option pricing; Black-Scholes equation.

1. Introduction

Black-Scholes model is an important milestone in modern finance. It was developed by Black and Scholes in 1973 [1]. Their pricing theory has a very close connection with the actual operation of the financial market and has been directly applied to the practice of financial transactions, which greatly promotes the rapid development of the global derivative financial market.

With the deepening of option pricing theory, on the basis of Black-Scholes model, scholars put forward Merton model [2], Heston model [3] and Bates model [4] to improve the theory of option pricing model. The numerical methods of option pricing mainly include Monte Carlo method [5], binary tree method [错误!未找到引用源。], finite difference method [7-10], radial basis function method [错误!未找到引用源。] and so on. Boyle [5] proposed the Monte Carlo simulation method for option pricing for the first time. He used the idea of Monte Carlo simulation to simulate the movement trajectory of stock price through random path and obtain option price through risk-free interest rate discount, but the effect was not very good. Brenna and Schwartz [7] applied the finite difference method to option pricing for the first time, and proposed an implicit difference method to price American options in the jump process. Cox and Ross et. al [6] obtained the numerical solution of American option price by using the binary tree method. This method is simple and intuitive, but the convergence speed is very slow. Andreasen and Andreasen [13] proposed an unconditionally stable alternative direction implicit operator splitting method with second-order accuracy for the European option problem, and applied fast Fourier transform to solve the algebraic system generated discretely, which greatly reduced the calculation time. Zhao and Davison [8] proposed a compact difference scheme for option pricing, which achieved higher accuracy. Cen and Le [9] presented robust finite difference methods for European and American option pricing. Kwon and Lee [10] proposed a time discrete-time method for implicit Crank-Nicolson with jump-diffusion model, in which the integral operator in the model was approximated by numerical quadrilles. Guo and Wang [14] proposed an unconditionally stable time splitting method for the nonlinear Black-Sholes equation, and proved the stability, positivity and convergence of the system. Patel and Mehta [11] proposed a high-order compacted difference scheme for solving the Black-Scholes equation based on polynomial interpolation, and compared the errors in Grank-Nicolson scheme, forward difference scheme and backward difference scheme in the time direction. They

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proposed that under a larger time step, the three compact schemes converge to the same precision. Company and Egorova et. al [12] studied the high-dimensional American option pricing problem based on the local radial basis function method and demonstrated the reliability of the method. More articles on option pricing refer to [15-错误!未找到引用源。].

Radial basis function (RBF) interpolation is a real meshless computing method, which does not need to generate regular meshes like the finite difference method. Based on this, we can simulate the solution of Black-Scholes equation at randomly generated points, which can get a better approximation of the actual option price. Three different multiquadric quasi-interpolations were constructed according to whether they are linear or depend on the derivative at the extreme value by Beaston and Powell [18], denoted as L_A, L_B, L_C. Then Wu and Schaback [19] proposed the quasi-interpolation operator L_D which was proved to be llinear and doesn't depend on the derivative at the extreme value. Wu and Chen [20] constructed a special operator L_D on this basis. More details on radial basis quasi-interpolation methods can be found in [21-24]. In this paper, the radial basis quasi-interpolation method is used to solve the Black-Scholes equation, which overcomes the disadvantage of the large number of conditions caused by the radial basis function interpolation, and also has a good fitting effect for solving partial differential equations.

The remaining work of this paper is as follows. In Section 2, multiquadric (MQ) quasi-interpolation operator and the Black-Scholes equation are introduced. In Section 3, the numerical scheme for solving European options with MQ quasi-interpolation operator and the convergence of the scheme are given. The results of numerical experiments are presented in Section 4. The conclusions are given in section 5.

2. Preliminarie

2.1 The Black-Scholes equation

In this paper, we consider the Black-Scholes equation

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + \left(r - \frac{1}{2}\sigma^2\right)\frac{\partial V}{\partial S} - rV = 0, \quad (S,t)\epsilon[0,+\infty) \times [0,T], \tag{2.1}$$

$$V(S,T) = \begin{cases} \max{(S-K,0)} & (for\ call), \\ \max{(K-S,0)} & (for\ put), \end{cases} \tag{2.2}$$

where V is the value of European option with the strike K and expiry date T, S is the price of stock, r is the risk-free rate, σ is the volatility. By using the algebraic transformation $\tau = T - t$, $x = lnS$ and $u(x,\tau) = V(x,t)$, we can get the new form of the Black-Scholes equation

$$\frac{\partial u}{\partial \tau} - \frac{1}{2}\sigma^2 \frac{\partial^2 u}{\partial x^2} - \left(r - \frac{1}{2}\sigma^2\right)\frac{\partial u}{\partial x} + ru = 0, \qquad (x,\tau)\epsilon(-\infty,+\infty) \times (0,T], \tag{2.3}$$

$$V(S,0) = \begin{cases} \max{(e^x - K,0)} & (for\ call), \\ \max{(K - e^x,0)} & (for\ put). \end{cases} \tag{2.4}$$

The solution $V(S,t)$ of the Black-Scholes equation (2.3) with the initial value (2.4) is

$$u(S,\tau) = \begin{cases} SN(d_1) - Ke^{-r\tau}N(d_2) & (for\ call), \\ Ke^{-r\tau}N(-d_2) - SN(-d_1) & (for\ put), \end{cases}$$

where

$$d_1 = \frac{ln\frac{S}{K} + (r + \frac{1}{2}\sigma^2)\tau}{\sigma\sqrt{\tau}}, \quad d_2 = d_1 - \sigma\sqrt{\tau}.$$

2.2 MQ quasi-interpolation

In this paper, we choose the MQ quasi-interpolation operator proposed by Wu and Chen [20] to approximate f.

Given the points (x_j, f_j), $\{f = f(x_j), j = 0,1,\dots,m, x_0 < x_1 < \cdots < x_m\}$, the MQ quasi-interpolation is defined as

$$f^*(x) = \sum_{j=0}^{m} f_j \Psi_j(x), \tag{2.5}$$

where