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Physical Informed Neural Network for Solving Conservation Laws Based on Relaxation Systems

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Abstract. Solving partial differential equations (PDEs) with discontinuous solutions, such as shock waves in multiphase viscous flows through porous media, is critical for a wide range of scientific and engineering applications. These discontinuities represent sudden changes in physical quantities. In recent years, physics-informed neural networks (PINNs) have emerged as a promising method for solving PDEs but face significant challenges when modeling such problems. Specifically, neural networks struggle to compute gradients accurately near shock waves, leading to solutions that deviate from true physical phenomena. To address this issue, we propose a novel relaxation neural network method based on the conservation law relaxation model and its improved version, the relaxation limit neural network method. These two approaches employ auxiliary neural networks to approximate flux functions, enhancing the vanilla PINN framework's capability to simulate shock waves. The proposed methods retain the simplicity and extensibility of vanilla PINNs while avoiding the need for spatiotemporal discretization. Numerical experiments for one-dimensional and two-dimensional problems demonstrate the effectiveness of our approaches. The results show that the improved methods significantly outperform vanilla PINN in capturing shock wave dynamics.

AMS subject classifications: 35L65, 65M99

Key words: Physics-informed neural networks, Partial differential equations, Conservation laws, Relaxation systems.

1 Introduction

Discontinuities in solutions to partial differential equations (PDEs) are ubiquitous in scientific and engineering applications. They arise from sudden changes in system properties, such as velocity, pressure, or phase volume fraction, and typically manifest as shock

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waves, contact discontinuities, or spikes. These phenomena occur in diverse contexts, including high-speed aerodynamics [1], astrophysical explosions [2], molecular transport in materials [3, 4], and multiphase flows through porous media [5, 6]. In porous media, discontinuities result from complex interactions among viscous, capillary, and inertial forces, abruptly altering fluid properties. Such behavior challenges traditional modeling and simulation approaches [7].

Mathematically, discontinuities in the solutions of PDEs typically arise from nonlinearities in the governing equations. For instance, in multiphase flow through porous media, the governing equations are derived from mass conservation laws and Darcy's law [8], which relates the volumetric flow rate to fluid viscosity, permeability, and pressure gradients. When viscous forces dominate such as under high-pressure gradients or in coarse grained porous materials—shock waves may develop. Due to the nonsmooth nature of these shocks, traditional discretization methods often introduce nonphysical oscillations or numerical diffusion, making such problems challenging to solve numerically. Conventional numerical techniques for PDEs, including finite difference (FD), finite element (FE), and finite volume (FV) methods, have been widely employed to model these systems [9]. However, accurately capturing discontinuities requires specialized shock-capturing schemes, such as total variation diminishing (TVD) methods, weighted essentially non-oscillatory (WENO) schemes, or adaptive mesh refinement (AMR). These approaches stabilize numerical solutions near discontinuities while preserving high-order accuracy in smooth regions. Despite decades of progress, balancing computational efficiency, accuracy, and robustness remains challenging, especially for complex multiphysics problems.

PINNs have recently emerged as a promising alternative for solving PDEs, utilizing the universal approximation capability of neural networks to directly encode physical laws into loss functions [10]. PINNs offer several advantages, including mesh-free discretization, seamless handling of high-dimensional problems, and the ability to incorporate experimental or observational data. However, vanilla PINNs struggle to resolve discontinuities effectively, often leading to inaccurate or oscillatory solutions near shocks [11, 12]. This limitation stems from the smoothness of neural network representations and the difficulty of optimizing the loss landscape in the presence of sharp gradients. De Ryck and Mishra [13] demonstrated that the estimation error of physics-informed machine learning methods can be significantly affected by the stability of the underlying PDEs (their sensitivity to perturbations). To address these challenges, several strategies have been proposed, including adaptive weighting of loss terms [14], domain decomposition [15], and artificial viscosity [16]. While each method provides unique advantages for addressing discontinuities, they still prove inadequate for complex problems like multiphase flow in porous media. Recent developments include the control volume PINN proposed by Patel et al. [17], which incorporates TVD conditions, and the weak PINN introduced by De Ryck et al. [18] for shock wave capture. Chaumet and Giesselmann [19] further developed a more efficient weak PINN scheme with entropy admissibility conditions to ensure solution uniqueness. However, these methods typi-