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Dealiased Seismic Data Interpolation Using Time Dynamic Warping with Dictionary Learning

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Abstract. In seismic data, interpolating regularly missing traces is generally regarded as more challenging than interpolating irregularly missing traces. To address regularly missing cases, an anti-aliasing strategy should be incorporated. In this paper, we employed dictionary learning approaches for seismic data anti-aliasing interpolation. In dictionary learning, it is crucial to pre-interpolate the sampled data to ensure that the learning dictionary captures the data structure. Currently, the nearest trace interpolation method is being used for pre-interpolation, which fails to utilize the spatial characteristics of data events. To overcome this limitation, we propose a pre-interpolation dictionary learning method based on time dynamic warping. The time dynamic warping technique calculates the similarity between two adjacent sampling traces and establishes the most similar path between the points. Subsequently, pre-interpolation data is obtained by linearly interpolating between these similar points. In the experimental comparison, we evaluate the performance of our proposed approach against the nearest pre-interpolation dictionary learning method. Synthetic and field date both demonstrate superior performance when using our proposed approach compared to the nearest pre-interpolation dictionary learning method.

AMS subject classifications: 68T05, 97N50

Key words: Seismic data interpolation, Dynamic time warping (DTW), Dictionary learning. †

1 Introduction

Due to economic limitations or environmental constraints, incomplete traces in obtained seismic data, whether irregular or regular along the spatial coordinates, are inevitable. This has a significant impact on seismic inversion, amplitude versus angle analysis, and migration. Therefore, interpolation plays a fundamental role in addressing this issue [1].

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Most state-of-the-art seismic interpolation algorithms, such as sparse transform, deep learning, and dictionary learning, have gained popularity in the past decade. Sparse transform methods, including wavelet [9], curvelet [10], and shearlet [1], enable sparse representation of seismic data in a transform domain. In the interpolation method proposed by Yang et al. [10], the integration of the curvelet transform into the projection onto convex sets(POCS) method aims to extract localized features from seismic data. Gan at al. [1] utilized the seislet transform to interpolate regularly missing traces in seismic data. However, sparse transform interpolation methods rely on the sparsity assumption and require different parameter selections for different datasets, resulting in poor flexibility. Recently, deep learning has emerged as a new algorithm that can automatically learn features and relationships hidden in large datasets. Wang at al. [8] applied advanced deep learning methods to perform anti-aliasing interpolation on seismic data, allowing for the extraction of complex features from the training data in a non-linear fashion using self-learning techniques. This methodology can effectively address the limitations posed by the linear assumptions, sparsity, and low-rank constraints that are commonly associated with traditional interpolation techniques. Saad et al. [7] proposed an unsupervised deep learning framework for simultaneous denoising and reconstruction of 3D seismic data, without the need for prior information or labels. Zhang et al. [13] designed an interpolation technique for seismic data utilizing denoising convolutional neural networks (CNNs). However, network construction in such methods relies on labeled data and faces challenges of overfitting. Dictionary learning (DL) is another adaptive learning method that does not require labeled data. Various types of dictionary learning have been proposed, such as K-means singular value decomposition (K-SVD) [6], data-driven tight frame (DDTF) [11], tree structure dictionary learning [2] and double learning method [4]. In these DL methods, K-SVD is one of the most effective overcomplete learning algorithms. The basic idea is to represent the dictionary as a matrix, where each column is represented by an atom (a basis in the dictionary). The algorithm optimizes the results by alternating the process of updating the dictionary and sparse representation. In each iteration, the atoms in the dictionary and the sparse representation of the data are continuously updated by minimizing the residual between the data and its sparse representation while maintaining the sparsity of the dictionary [3]. Since one SVD decomposition only updates one column in the dictionary, multiple SVD decompositions need to be performed in each iteration, resulting in a large amount of calculation. Based on this situation, the DDTF method [11] is proposed. This method only needs to update the dictionary through one SVD operation in each iteration, which significantly reduces computational requirements and enhances efficiency in calculations.

In DDTF method, the dictionary is learned from the sampled data. However, since the sampled data contain missing traces, the DL method cannot capture the data structure effectively. Therefore, pre-interpolation is essential before dictionary learning to restore the missing traces. The nearest trace interpolation (NNI) method was used by [2] and [11]. NNI simply utilizes the nearest trace to estimate the missing trace, without considering the underlying data structure. Consequently, the resulting dictionary lacks the necessary data detail features. A reliable pre-interpolation method is therefore crucial for the dictionary learning process and has a significant impact on the quality of interpolation.

This letter presents a pre-interpolation method utilizing dynamic time warping (DTW) [5,12]. DTW aims to find the optimal path that matches two time series, enabling a comparison of their similarity. This path aligns the time points and minimizes the overall distance between the sequences. Based on this shortest path, we identify columns of similar points in the two nearest sampled traces, calculate the slopes between these points, and utilize these slopes for linear interpolation in the corresponding direction. During the seismic interpolation testing, we compare the performance of DTW pre-interpolation with that of nearest pre-interpolation within dictionary learning. The results reveal that the DTW method got better interpolation outcomes.

2 Method

Reconstructing the complete dataset Y from observed incomplete data X is the primary objective of seismic data interpolation. This process can be elucidated as

$$X = AY, \tag{2.1}$$

where A is a downsampling matrix. It is a widely known fact that any matrix can be expressed as a combination of a dictionary and its corresponding coefficients in the dictionary domain. This can be expressed as

$$Y = D_{Dic}C,$$

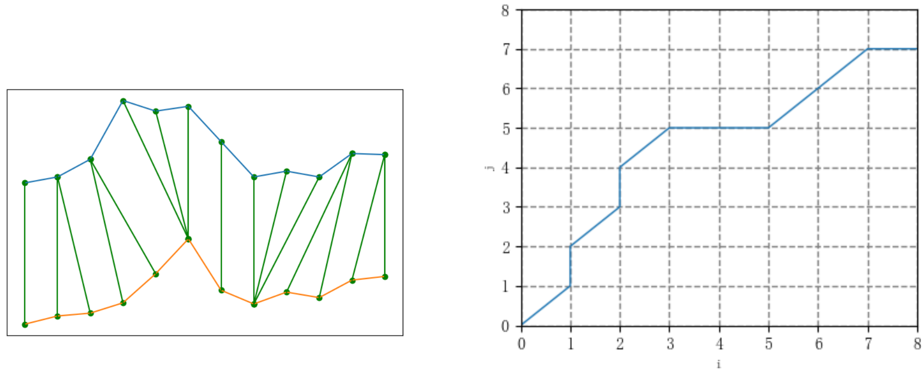
wheret D_{Dic} is a dictionary, and C is coefficients matrix. Therefore, we can reconstruct the data Y using a dictionary learning optimization problem

$$\underset{D_{Dic},C}{\operatorname{argmin}} ||Y - D_{Dic}C||_F^2 + \lambda ||C||_0, \quad s.t. D_{Dic}^T D_{Dic} = I, \tag{2.2}$$

where $||\cdot||_F$ denotes the Frobenius norm, which is the matrix's 2-norm; $||\cdot||_0$ represents the L_0 norm, indicating the number of nonzero elements, λ is a regularization parameter. The interpolation data Y in eq.2.2 is obtained by performing two alternating steps: dictionary learning and coefficient learning.

2.1 In the dictionary learning step

We need to learn the dictionary from the sampled data X. However, the data X contains missing traces. Learning features of the data becomes difficult if we directly learn dictionaries from missing data. Therefore, pre-interpolation of the data is necessary before learning. In the past, pre-interpolation was done using nearest trace interpolation. This method fails to utilize the spatial characteristics of the data, and pre-interpolation



(a) (b)

Figure 1: (a) Dynamic time warping between two time series, (b) A cost matrix displaying the warp path with the minimum distance.

does not solve the issue of seismic data aliasing due to regular sampling. To address this problem, we introduce a pre-interpolation method that utilizes dynamic time warping.

Dynamic time warping measures the similarity between two time series (see Fig.1a). In Fig.1a, the upper and lower solid lines represent two time series, x_1 and x_3, respectively, whereas the line between two points indicates the corresponding points found to be similar between the two time series. The DTW process can be divided into two main components: constructing the distance matrix and identifying the shortest path. The process can be expressed as follows.

1. Initialize $Dist(1,1)=0$, indicating that the first elements of sequences x_1 and x_3 have a distance of 0.

2. Calculate the distance between each element, $Dist(i,j)$, and its neighboring elements above, to the left, and diagonally upper-left in the $Dist$ matrix. Select the minimum distance among these neighbors as the minimum path distance for the current element. In other words,

$$Dist(i,j)=Dist(i,j)+\min[Dist(i-1,j),Dist(i,j-1),Dist(i-1,j-1)].$$

3. Repeat step 2 until the cumulative distance for all elements has been calculated. The final element of the cumulative distance matrix, Dist(x_1,x_3), represents the dynamic time warping distance between sequence x_1 and sequence x_3. By backtracking along the minimum path in the cumulative distance matrix, Dist(x_1,x_3), we can obtain the corresponding warping path (see Fig. 1b).

4. Using the shortest path, we identify the columns in x_1 and x_3 that contain similar points. For one-to-one similar points, we compute the slopes between these similar

points and utilize them for linear interpolation in the corresponding directions. In the case of one-to-many similar points, we select the three closest points and compute the slopes between these similarities individually, taking the average of them for linear interpolation. In this process, the pre-interpolated data x^2_{pre1} is generated. And repeat the aforementioned steps for x^2_{pre1} and x_3 to obtain pre-interpolated data x^2_{pre2}. Ultimately, x^2_{pre1} and x^2_{pre2} are weighted to derive the final pre-interpolated data x^2_{pre}. The formula is

$$x^2_{pre} = w_0 * x^2_{pre1} + (1 - w_0) * x^2_{pre2}.$$

For the missing data X, we perform step 1 to step 4 and get the pre-interpolation data X_{pre}. The DDTF algorithm [11] is applied to learn the dictionary from the data X_{pre}. The learned dictionary can be represented as $D_{Dic} = SVD(X_{pre}C^T)$, where SVD refers to singular value decomposition.

2.2 In the coefficient step

The DDTF method is employed to obtain the coefficients by applying a hard thresholding operator, denoted as H, to the tight frame coefficients $C = H(D^T_{Dic} X_{pre})$. Consequently, the interpolation data, Y_{rec}, is obtained as $Y_{rec} = D_{Dic}C$.

3 Experimental

In this section, we conduct a comparison between two algorithms for seismic interpolation: nearest pre-interpolation dictionary learning (NDL) and dynamic time warping pre-interpolation dictionary (DTWDL). Our experiments involve interpolation using regular missing and irregular missing of synthetic data and field data respectively. Except for the synthetic data with irregular missing, which is 512∗256, the other data sizes are all 512∗512. The NDL method utilizes the nearest trace for pre-interpolation. When the left and right traces are equidistant from the missing trace, we select the left trace for pre-interpolation. On the other hand, DTWDL employs dynamic time warping to calculate the distance between adjacent sampling traces and obtain the alignment path. Based on this alignment path, the slope of similar points in the two tracks is calculated, and linear interpolation is then used to fill in the missing trace.

The signal-to-noise ratio (S/N), which is employed to assess the quality of the restoration, is described as

$$S/N = 10\log_{10}\left(\frac{\|Y\|_F^2}{\|Y - X_{rec}\|_F^2}\right).$$

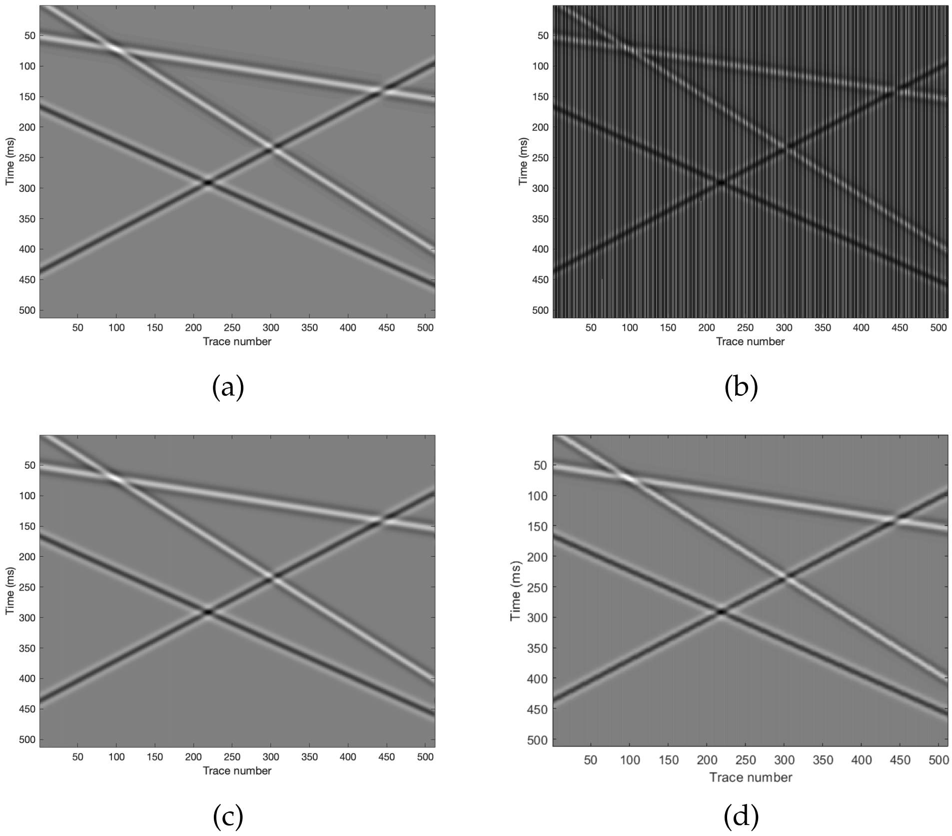


Figure 2: Synthetic data one. (a) Original seismic data, (b) Sampling data, (c) NDL, (d) DTWDL.

3.1 2D data

Initially, we showcase the efficiency of our approach in pre-interpolating regular missing synthetic data. Fig.2a illustrates the synthetic data featuring three events. The 50% regular sub-sampled data is shown in Fig.2b. The interpolated result using our proposed DTWDL method is depicted in Fig.2d. As shown in Table.1, yielding a recovered S/N ratio of 44.8492 dB. To contrast, we display the recovery outcome achieved with NDL, which achieves an S/N ratio of 36.3976 dB, in Fig.2c. To further compare the different pre-interpolation methods, Fig.3c and Fig.3d show the results obtained from Nearest pre-interpolation and DTW pre-interpolation, respectively, Its S/N ratio is shown in Table1. Fig.3e and Fig.3f represent the spectra of Nearest pre-interpolation in Fig.3b and proposed DTW pre-interpolation in Fig.3c. Spatial aliasing is observed in Fig.3e, and it is well-removed by the proposed method, as shown in Fig.3f. Moreover, Fig.4 illustrates the reconstruction errors. A comparison between Fig.4a and Fig.4b clearly reveals that the DTW pre-interpolated result captures more data features, as denoted by the red arrow. Thus, better dictionary learning can be achieved using the DTW pre-interpolation. Fig.4c and Fig.4d display the corresponding reconstruction errors for Fig.2c and Fig.2d, respectively. Reconstruction residual of our method has a very small magnitude.

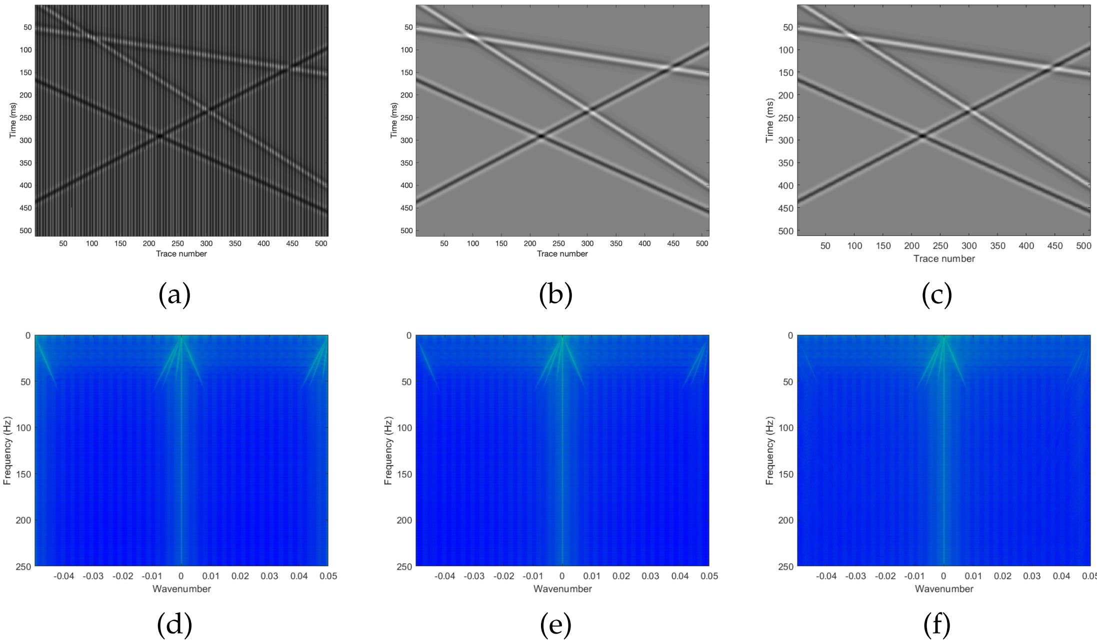


Figure 3: Synthetic data. (a) Sampling data. (b) Nearest pre-interpolation(S/N=36.8823). (c) DTW pre-interpolation(S/N=49.4134). (d) The $f-k$ spectra of the data in Fig.3a. (e) The $f-k$ spectra of the data in Fig.3b. (f) The $f-k$ spectra of the data in Fig.3c.

To further demonstrate the versatility of the proposed DTWDL method, we utilize a field data example, as depicted in Fig.5. In Fig.5a-5f, a comparative analysis is presented showcasing the interpolated outcomes from NDL and DTWDL using seismic trace visualization. Comparisons of the Nearest pre-interpolation and DTW pre-interpolation are depicted in Fig.5c and Fig.5d. Our proposed DTWDL method yields an interpolated seismic trace that closely approximates the original seismic trace.

To illustrate the specifics of the interpolation process, we extract four traces from the reconstructed data corresponding to the missing traces of the 22nd, 244th, 404th, and 502nd traces, as depicted in Fig.6. The original data is represented by black curves, NDL interpolation data by blue dashed lines, and DTWDL interpolation data by red dashed lines. Analysis of the results shown in Fig.6 reveals that the DTWDL method exhibits higher interpolation accuracy than the NDL method. As a result, the interpolation error associated with the DTWDL method is smaller compared to that of the NDL method. The effectiveness of our method in pre-interpolating synthetic data is also demonstrated in irregularly missing data. Fig.7a illustrates synthetic data with three events. The 50% irregular sub-sampling data is shown in Fig.7b. The results of inter-

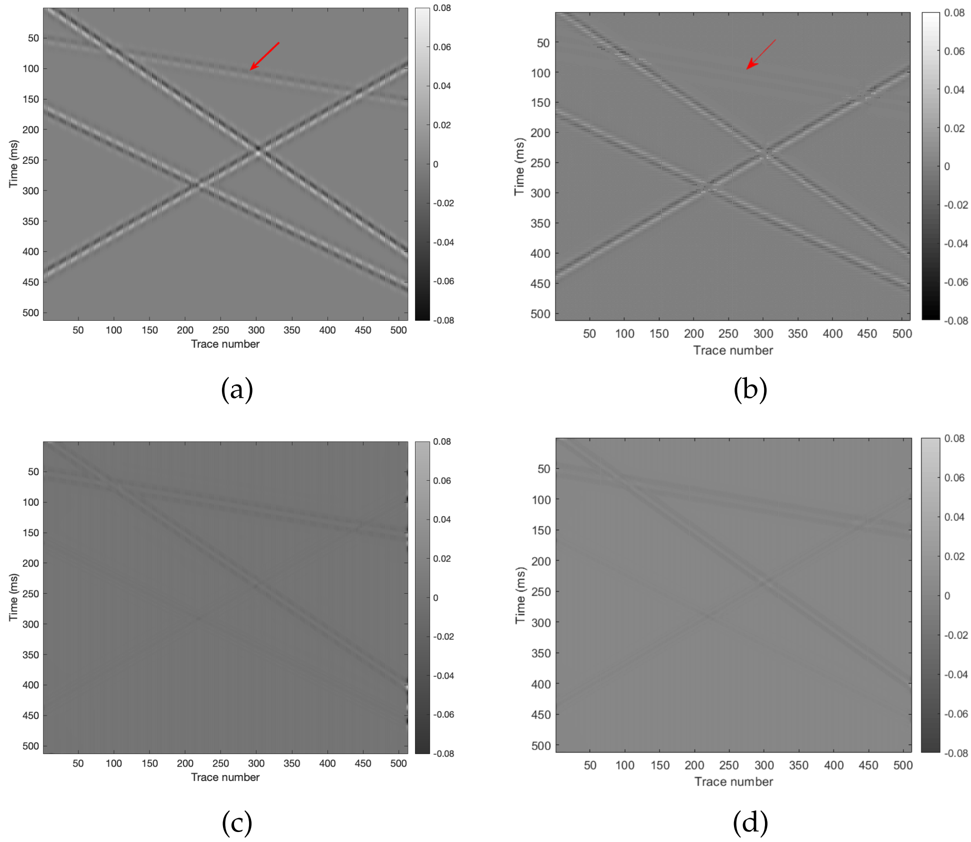


Figure 4: Reconstruction errors. (a) Nearest pre-interpolation, (b) DTW pre-interpolation, (c) NDL, (d) DTWDL.

polation using our proposed DTWDL method are shown in Fig.7f, and the recovered S/N ratio is 35.6058 dB. The reconstruction result of NDL is shown in Fig.7e. Additionally, Figure 8 also illustrates the reconstruction error. Therefore, applying DTW pre-interpolation to data sets characterized by irregular missing patterns can still result in enhanced dictionary learning performance. Likewise, in order to show the interpolation of the DTWDL method. Fig.9b shows 50% irregularly subsampled data. Interpolation results for the Nearest pre-interpolation, DTW pre-interpolation, NDL and DTWDL methods(S/Ns 26.4573, 28.9350, 30.7564 and 31.3948 dB, respectively) are displayed in Fig.9. Overall, the performance of these four methods in recovering results is highly satisfactory. The performance of the DTWDL method get more coherent in seismic events.

3.2 3D data

Additionally, we showcase the effectiveness of the DTWDL method on 3D data. Fig.10a shows the original 3D data(Its size is 876∗221∗271.). The 50% regular sub-sampling data

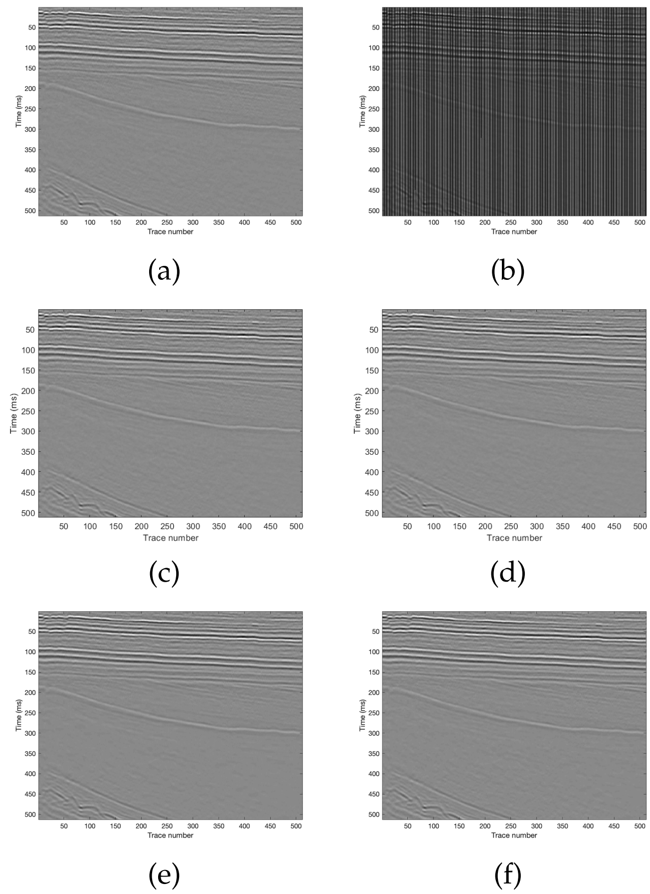


Figure 5: Field data one. (a) Original seismic data, (b) Sampling data, (c) Nearest pre-interpolation, (d) DTW pre-interpolation, (e) NDL, (f) DTWDL.

is shown in Fig.10b. The result of interpolation using our proposed DTWDL method is shown in Fig.10d, and the recovered S/N ratio is 12.2057 dB. The reconstruction result of NDL is shown in Fig.10c, with a S/N ratio of 11.6601 dB.

In order to facilitate a more intuitive observation, the 75th slice in the 3D data was chosen, as depicted in Fig.11. The interpolated outcomes obtained through our DTWDL method are displayed in Fig.11d, yielding a restored S/N ratio of 11.7420 dB. The reconstruction results of NDL are presented in Fig.11c, showing a S/N ratio of 9.7311 dB. Hence, the application of DTW pre-interpolation on 3D datasets featuring regular missing patterns can still improve the efficiency of dictionary learning.

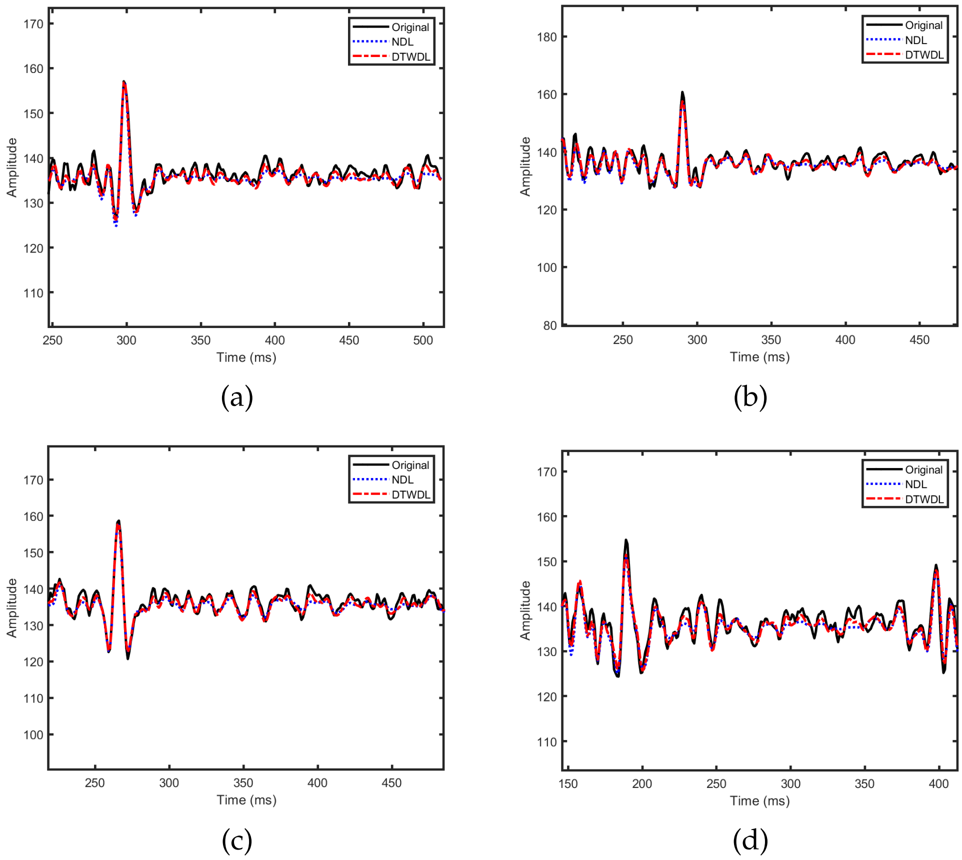


Figure 6: Trace comparison. The solid black line corresponds to the original trace, and the blue dotted and red dotted lines represent the reconstructed trace by NDL and DTWDL. (a) 502th trace, (b) 404th trace, (c) 244th trace, (d) 22th trace.

4 Conclusion

We introduce a novel method, namely DTWDL, for seismic data interpolation. In this approach, DTW is utilized to calculate the similarity between the sampled points in the nearest trace, resulting in the pre-interpolation stage. Then we learned dictionary from the pre-interpolation data and get the interpolation data. In the experiments section, we give the test results of the interpolation through the DTWDL and NDL. DTWDL achieves state-of-the-art performance in data interpolation. In the furture, we would like to make the learning algorithm robust to big gap data interpolation.

Table 1: Correlated S/N for regularly missing seismic data interpolation.

method	Synthetic data one	Field data one
Nearest pre-interpolation	36.8823	36.4315
DTW pre-interpolation	41.7986	39.1178
NDL	36.3976	36.9294
DTWDL	44.8492	38.5850

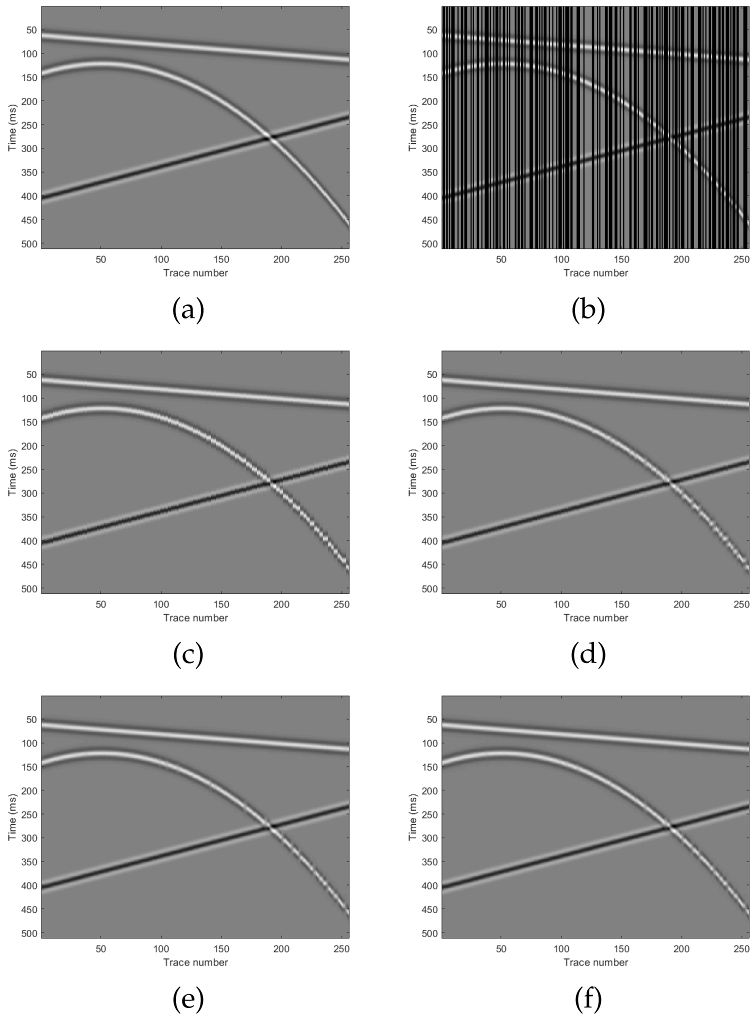


Figure 7: Synthetic data two. (a) Original seismic data, (b) Sampling data, (c) Nearest pre-interpolation, (d) DTW pre-interpolation, (e) NDL, (f) DTWDL.

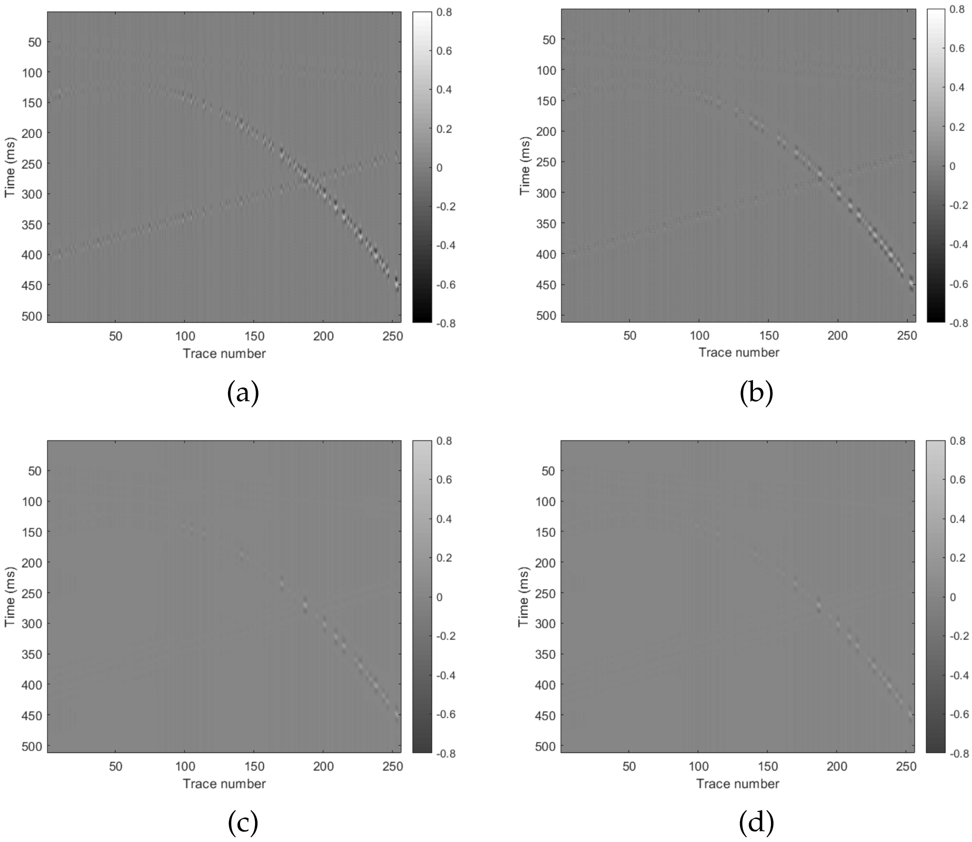


Figure 8: Reconstruction errors. (a) Nearest pre-interpolation, (b) DTW pre-interpolation, (c) NDL, (d) DTWDL.

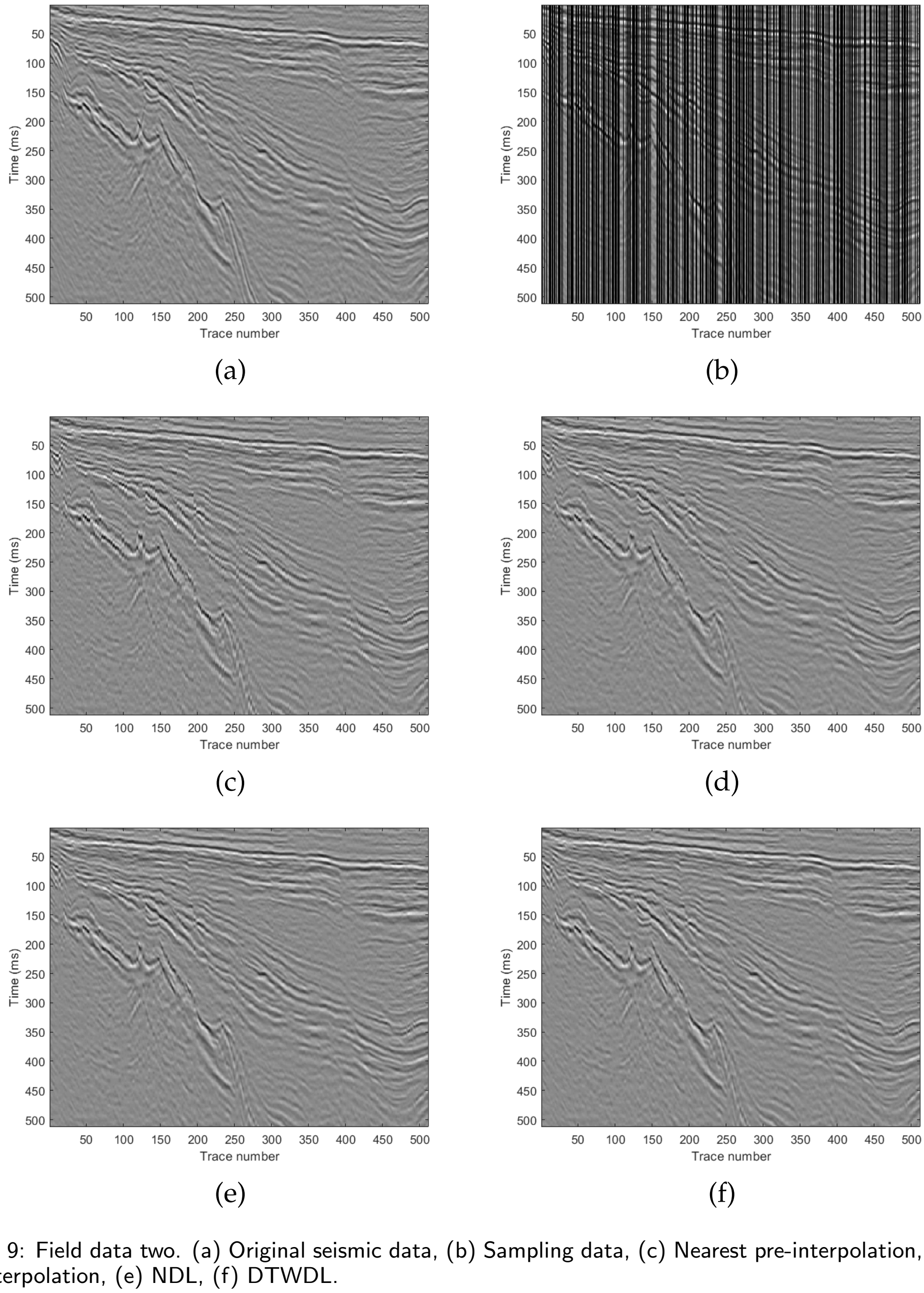


Figure 9: Field data two. (a) Original seismic data, (b) Sampling data, (c) Nearest pre-interpolation, (d) DTW pre-interpolation, (e) NDL, (f) DTWDL.

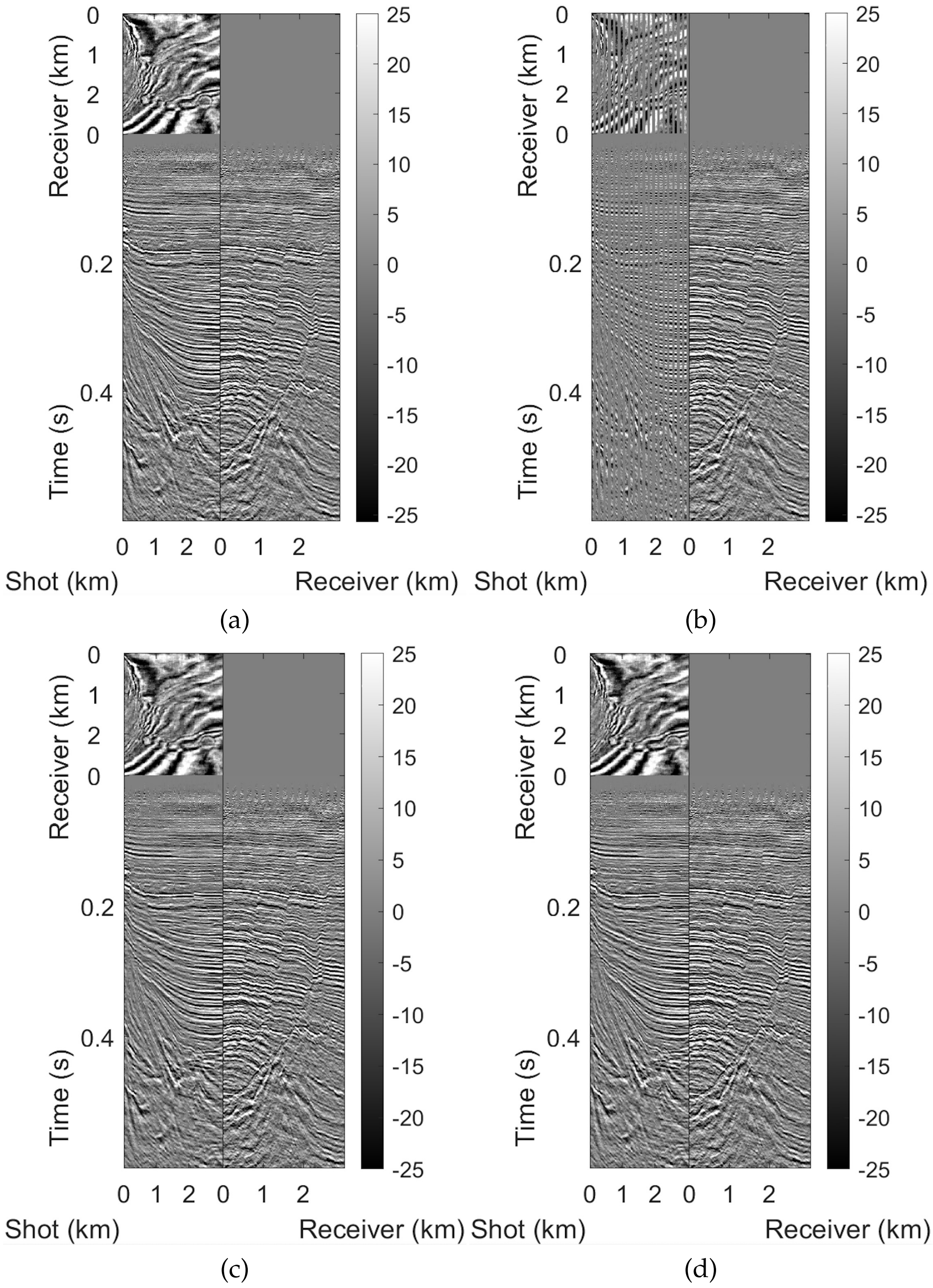


Figure 10: 3D Field data. (a) Original seismic data, (b) Sampling data, (c) NDL (S/N=11.6601), (d) DTWDL (S/N=12.2057).

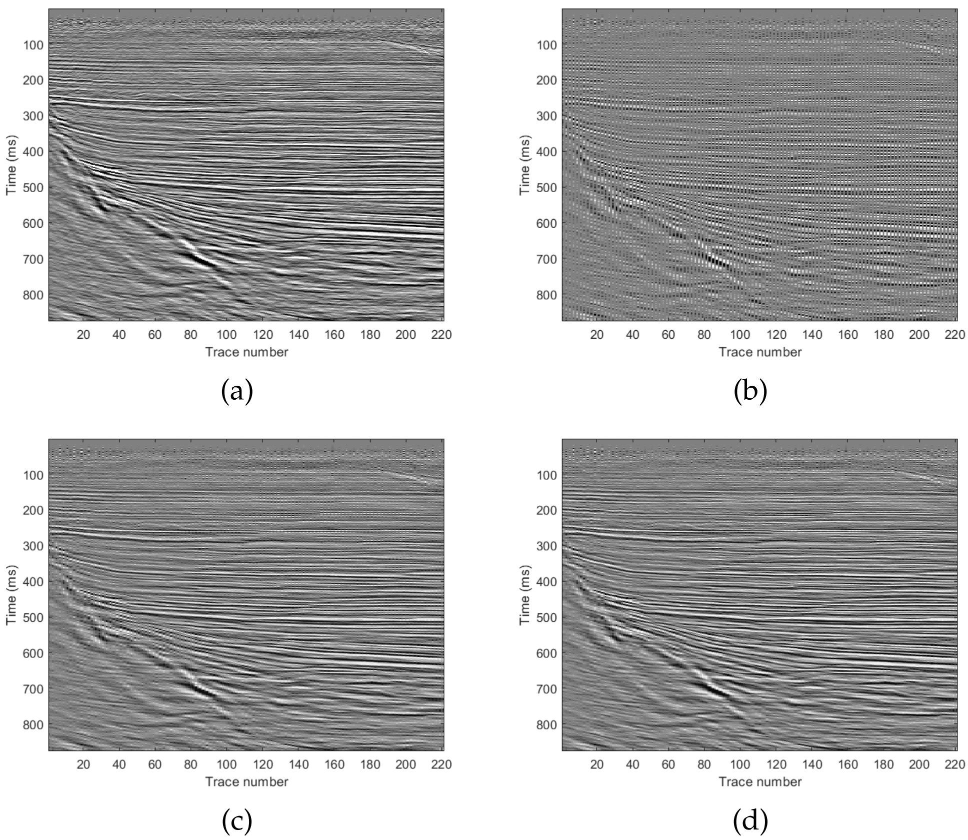


Figure 11: The 75th slice within the 3D field data. (a) Original seismic data, (b) Sampling data, (c) NDL (S/N=9.7311), (d) DTWDL (S/N=11.7420).

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