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# Neural Stochastic Volterra Equations: Learning Path-Dependent Dynamics

Martin Bergerhausen * [1], David J. Prömel † [1], and David Scheffels ‡ [1]

[1]University of Mannheim, Mannheim 68161, Germany.

**Abstract.** Stochastic Volterra equations (SVEs) serve as mathematical models for the time evolutions of random systems with memory effects and irregular behaviour. We introduce neural stochastic Volterra equations as a physics-inspired architecture, generalizing the class of neural stochastic differential equations, and provide some theoretical foundation. Numerical experiments on various SVEs, like the disturbed pendulum equation, the generalized Ornstein-Uhlenbeck process, the rough Heston model and a monetary reserve dynamics, are presented, comparing the performance of neural SVEs, neural stochastic differential equations (SDEs) and Deep Operator Networks (DeepONets).

**Keywords:**
Feedforward neural network,
Deep operator network,
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## 1 Introduction

Stochastic Volterra equations are used as mathematical models for the time evolutions of random systems appearing in various areas like biology, finance or physics. SVEs are a natural generalization of ordinary stochastic differential equations and, in contrast to SDEs, they are capable to represent random dynamics with memory effects and very irregular trajectories. For instance, SVEs are used in the modelling of turbulence [2], of volatility on financial markets [12] and of DNA patterns [36].

Combining differential equations and neural networks into hybrid approaches for statistical learning has been gaining increasing interest in recent years, see e.g. [5,11]. This has led to many very successful data-driven methods to learn solutions of various differential equations. For instance, neural stochastic differential equations are SDEs with coefficients parametrized by neural networks, and serve as continuous-time generative models for irregular time series, see [15, 19, 26, 27]. Models based on neural SDEs are of particular interest in financial engineering, see [7, 9, 13]. Further examples of "neural" differential equations are neural controlled differential equations [21], which led to very successful

*martin.bergerhausen@uni-mannheim.de
†Corresponding author. proemel@uni-mannheim.de
‡david.scheffels@lessing-ffm.net

methods for irregular time series, neural rough differential equations [30], which are especially well-suited for long time series, and neural stochastic partial differential equations [37], which are capable to process data from continuous spatiotemporal dynamics. Loosely speaking, "neural" differential equations and their variants can be considered as continuous-time analogous to various recurrent neural networks.

In the present work, we introduce neural stochastic Volterra equations as stochastic Volterra equations with coefficients parameterized by neural networks. They constitute a natural generalization of neural SDEs with the advantage that they are capable to represent time series with temporal dependency structures, which overcomes a limitation faced by neural SDEs. Hence, neural SVEs are suitable to serve as generative models for random dynamics with memory effects and irregular behaviour, even more irregular than neural SDEs. As theoretical justification for the universality of neural SVEs, we provide a stability result for general SVEs in Proposition 2.1, which can be combined with classical universal approximation theorems for neural networks [10, 14, 21, 25]; cf. Remark 2.1.

Relying on neural stochastic Volterra equations parameterized by feedforward neural networks, we study supervised learning problems for random Volterra type dynamics. More precisely, we consider setups, where the training sets consist of sample paths of the "true" Volterra process together with the associated realizations of the driving noise and the initial condition, and build a neural SVEs based model aiming to reproduce the sample paths as good as possible. A related supervised learning problem in the context of stochastic partial differential equations (SPDEs) was treated in [37] introducing neural SPDEs. For unsupervised learning problems using neural SDEs we refer to [18].

We numerically investigate the supervised learning problem for prototypical Volterra type dynamics such as the disturbed pendulum equation [31], the rough Heston model [12], the generalized Ornstein-Uhlenbeck process [40] and a model for the dynamics of monetary reserves [3]. The performance of the neural SVE based models is compared to Deep Operator Networks and to neural SDEs. Recall DeepONets are a popular class of neural learning algorithms for general operators on function spaces that were introduced in [28]. For the training process of the neural SVE we choose the Adam algorithm, as introduced in [22], which is known to be a well-suited stochastic gradient descent method for stochastic optimization problem.

The numerical study in Section 3 demonstrates that the presented neural SVE based methods significantly outperform DeepONets; see Tables 3.1-3.3. In particular, neural SVE based methods generalize much more effectively, as evidenced by their strong performance on the test sets – neural SVEs are up to 20 times more accurate than DeepONets. Moreover, neural SVEs also outperform neural SDE based models for random dynamics with dependency structures; cf. Section 3.5. These observations highlight the advantages of the physics-informed architecture of neural SVEs for supervised learning problems involving random systems with Volterra-type dynamics.

**Organization of the paper.** In Section 2 we introduce neural stochastic Volterra equations and their theoretical background. The numerical experiments are presented in Section 3. In Appendix A we present the postponed proofs regarding the stability of stochastic Volterra equations.