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Non-Traveling Wave Solutions of (3+1)-Dimensional Variable Coefficients BLMP Equation with a Compound Technology

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Abstract. We investigate non-traveling wave solutions of the (3+1)-dimensional Boiti-Leon-Manna-Pempinelli equation with time-dependent coefficients, which describes the propagation of waves in incompressible fluids. We creatively combine the extended three-wave method with the generalized variable separation method and successfully obtain sixty exact non-traveling solutions including kink-like solutions, singular solitary wave-like solutions, periodic solitary wave-like solutions, periodic kink-like solutions, periodic cross-kink-like waves, homoclinic breather wave-like solutions and so on. The variable coefficients and arbitrary functions in the obtained solutions are easy to exhibit abundant soliton structures, which may be of great significance for explaining some practical physical phenomena. By contour plots, 2D plots, and 3D plots, we analyze the dynamic characteristics of periodic cross-kink-like solution, singular solitary wave-like solution, homoclinic breather wave-like solution. Additionally, we show changes of solutions under different tails to illustrate the influence of tails on solutions.

AMS subject classifications: 35C99, 35G20, 37K10, 68W30

Key words: VC-BLMP equation, non-traveling solutions, extended three-wave method, generalized variable separation method.

1 Introduction

Nonlinear partial differential equation (NLPDE) is an important branch of modern mathematics, which is used to describe the problems in the fields of mechanics, control process, ecological and economic systems, chemical circulatory systems and epidemiology.

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Many high-dimensional or variable coefficients nonlinear partial differential equations which can better reflect complex and practical physical phenomena are established in recent years. Moreover, exact solutions can profoundly explain the physical model itself and predict the evolution process of the actual physical state. Therefore, seeking the explicit and accurate solutions of these equations is particularly important for further studying the dynamic processes described by these models.

Recently, many mathematicians and physicists have focused on exact solutions of nonlinear partial differential equations [1–3], especially non-traveling wave solutions [4–7, 9]. Ma [1, 2] used the direct reduction method and Hirota type ansatz of separation variables to get a large number of exact solutions for restricted Boiti-Leon-Pempinelli DLW system and (2+1)-dimensional equations. Reference [3] investigated integrability, bilinearization, soliton solutions, exact three wave solutions for a forced Korteweg-de Vries equation via Painlevé analysis, bilinear Bäcklund transformation, simplified Hirota method and exact three wave method. Based on the Hirota method, Reference [10] applied the quadratic functions to generate lump solutions of the (3+1)-dimensional generalized Calogero-Bogoyavlenskii-Schiff equation. Additionally, References [11–16] have investigated exact solutions of high dimensional partial differential equations with variable coefficients. It follows that deriving the exact non-traveling wave solutions of nonlinear partial differential equation with high dimensional variable coefficients have an important values both for researches and applications. As an extension of shallow water wave model in higher dimensions, (3+1)-dimensional Boiti-Leon-Manna-Pempinelli equation with constant coefficients

$$u_{yt}+u_{zt}+u_{xxxy}+u_{xxxz}-3u_x(u_{xy}+u_{xz})-3u_{xx}(u_y+u_z)=0 \tag{1.1}$$

describes the propagation of waves in incompressible fluids, the evolution of ion sound waves and internal waves in stratified oceans of incompressible media. In addition, when $z=0$, it describes the interaction between the Riemann wave propagating along the y-axis and the long wave propagating along the x-axis. The (3+1)-dimensional BLMP equation with constant coefficients describes ideal cases under many assumptions. However, if the inhomogeneity of the media, the inconsistency of the boundary, the variable depth, the existence of vortices and viscosity are considered in practical physical problems, a class of (3+1)-dimensional BLMP equation with time-dependent coefficients

$$f(t)(u_y+u_z)_t+g(t)(u_y+u_z)_{xxx}+h(t)(u_x(u_y+u_z))_x=0 \tag{1.2}$$

can be derived, where $f(t)$, $g(t)$ and $h(t)$ are non-zero functions of t. For $f(t)$=1, $g(t)=1$, $h(t)=-3$, Eq. (1.2) reduces to (3+1)-dimensional Boiti-Leon-Manna-Pempinelli equation with constant coefficients (1.1). Many researchers have investigated exact solutions of (1.1). Liu et al. [7] applied (G'/G)-expansion method to get a family of exact non-traveling wave solutions of (1.1). Reference [17] investigated the integrability of this equation and derived multiple real and complex soliton solutions of (1.1) by Hirota's direct method. Using bilinear forms under certain conditions, Hirota's direct method