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Intrinsic Enumerative Mirror Symmetry: Takahashi's Log Mirror Symmetry for $(\mathbb{P}^2,E)$ Revisited

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This article is dedicated to Gang Tian, who introduced the third author to quantum cohomology in 1993 and to so many other things, at the occasion of his 65th birthday.

Abstract. Let E be a smooth cubic in the projective plane $\mathbb{P}^2$. Nobuyoshi Takahashi formulated a conjecture that expresses counts of rational curves of varying degree in $\mathbb{P}^2\setminus E$ as the Taylor coefficients of a particular period integral of a pencil of affine plane cubics after reparametrizing the pencil using the exponential of a second period integral.

The intrinsic mirror construction introduced by Mark Gross and the third author associates to a degeneration of $(\mathbb{P}^2,E)$ a canonical wall structure from which one constructs a family of projective plane cubics that is birational to Takahashi's pencil in its reparametrized form. By computing the period integral of the positive real locus explicitly, we find that it equals the logarithm of the product of all asymptotic wall functions. The coefficients of these asymptotic wall functions are logarithmic Gromov-Witten counts of the central fiber of the degeneration that agree with the algebraic curve counts in $(\mathbb{P}^2,E)$ in question. We conclude that Takahashi's conjecture is a natural consequence of intrinsic mirror symmetry. Our method generalizes to give similar results for log Calabi-Yau varieties of arbitrary dimension.

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