Journal of Nonlinear Modeling and Analysis http://jnma-online.com
Volume 6, Number 2, June 2024, 265–272 DOI:10.12150/jnma.2024.265

The Upper Semicontinuity of Random Attractors for Non-Autonomous Stochastic Plate Equations with Multiplicative Noise and Nonlinear Damping*

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Abstract Based on the existence of pullback attractors for the non-autonomous stochastic plate equations with multiplicative noise and nonlinear damping defined in the entire space $\mathbb{R}^n$ by Xiaobin Yao in [15], in the paper, we further investigate the upper semicontinuity of pullback attractors for the problem.

Keywords Upper semicontinuity, attractors, plate equation, unbounded domains, multiplicative white noise

MSC(2010) 35B40, 35B41.

1. Introduction

Plate equations have been investigated for many years due to their importance in some physical areas such as vibration and elasticity theory of solid mechanics. The study of the long-time dynamics of plate equations has become an outstanding topic in the field of the infinite dimensional dynamical system [3–6, 9].

In this paper, we study the upper semicontinuity of pullback attractors for the following non-autonomous stochastic plate equation with multiplicative noise and nonlinear damping defined on the unbounded domain $\mathbb{R}^n$:

$$u_{tt} + \Delta^2 u + h(u_t) + \lambda u + f(x, u) = g(x, t) + \varepsilon u \circ \frac{dw}{dt} \tag{1.1}$$

with the initial value conditions

$$u(x, \tau) = u_0(x), \quad u_t(x, \tau) = u_1(x), \tag{1.2}$$

where $x \in \mathbb{R}^n$, $t > \tau$ with $\tau \in \mathbb{R}$, $\lambda > 0$ and ε are constants, $h(u_t)$ is a nonlinear damping term, f is a given interaction term, g is a given function satisfying $g \in L^2_{loc}(\mathbb{R}, H^1(\mathbb{R}^n))$, and w is a two-sided real-valued Wiener process on a probability space. The stochastic equation (1.1) is understood in the sense of Stratonovich's integration.

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*The authors were supported by National Natural Science Foundation of China (12161071) and Qinghai Minzu University-level innovation project (07 M 2023009).

The functions $h,\ f$ satisfy the following conditions.

(1) Let $F(x,u) = \int_0^u f(x,s)ds$ for $x \in \mathbb{R}^n$ and $u \in \mathbb{R}$. There exist positive constants $c_i (i = 1, 2, 3, 4)$, such that

$$|f(x,u)| \le c_1|u|^\gamma + \phi_1(x), \quad \phi_1 \in L^2(\mathbb{R}^n), \tag{1.3}$$

$$f(x,u)u - c_2F(x,u) \ge \phi_2(x), \quad \phi_2 \in L^1(\mathbb{R}^n), \tag{1.4}$$

$$F(x,u) \ge c_3|u|^{\gamma+1} - \phi_3(x), \quad \phi_3 \in L^1(\mathbb{R}^n), \tag{1.5}$$

$$|\frac{\partial f}{\partial u}(x,u)| \le \beta, \quad |\frac{\partial f}{\partial x}(x,u)| \le \phi_4(x), \quad \phi_4 \in L^2(\mathbb{R}^n), \tag{1.6}$$

where $\beta > 0$ and $1 \le \gamma \le \frac{n+4}{n-4}$.

(2) There exist two constants β_1, β_2 such that

$$h(0) = 0, \qquad 0 < \beta_1 \le h'(v) \le \beta_2 < \infty. \tag{1.7}$$

(3)

$$\delta > 0 \text{ satisfies } \ \lambda + \delta^2 - \beta_2\delta > 0,\ \beta_1 > \delta. \tag{1.8}$$

Just for problems (1.1)-(1.2) and the corresponding plate equations, on the unbounded domain, the authors investigated the asymptotic behavior for stochastic plate equation with different noise (see [12–15] for details). To the best of our knowledge, it has not been considered by any predecessors for the upper semicontinuity of pullback attractors for the stochastic plate equation with multiplicative noise on unbounded domain. It is well known that multiplicative noise makes the problem more complex and interesting even to the case of bounded domain. Based on the results in [15] as well as the theory and applications of B. Wang in [10,11], we decide to study the upper semicontinuity of pullback attractors for problems (1.1)-(1.2).

The rest of this paper is organized as follows. In the next section, we present some notations, definitions and a criteria concerning the upper semicontinuity of non-autonomous random attractors with respect to a parameter. In Section 3, we show the upper semi-continuity of random attractors.

Throughout the paper, we use $||\cdot||$ and $(\cdot,\cdot)$ to denote the norm and the inner product of $L^2(\mathbb{R}^n)$, respectively. The norms of $L^p(\mathbb{R}^n)$ and a Banach space X are generally written as $||\cdot||_p$ and $||\cdot||_X$, respectively. The letters c and c_i $(i = 1, 2, \dots)$ are generic positive constants which may change their values from line to line or even in the same line and do not depend on ε.

2. Preliminaries

In this section, we first present some notations, then recall some definitions and known results regarding non-autonomous random dynamical systems from [1,2,7, 8,11,16], which are useful to our problem.

Let $(\Omega, \mathcal{F}, \mathcal{P})$ be the standard probability space, where $\Omega = \{\omega \in C(\mathbb{R},\mathbb{R}) : \omega(0) = 0\}$, $\mathcal{F}$ is the Borel σ-algebra induced by the compact open topology of Ω, and $\mathcal{P}$ is the Wiener measure on $(\Omega, \mathcal{F})$. There is a classical group $\{\theta_t\}_{t\in\mathbb{R}}$ acting on $(\Omega, \mathcal{F}, \mathcal{P})$ which is defined by

$$\theta_t\omega(\cdot) = \omega(\cdot + t) - \omega(t), \quad \text{for all } \omega \in \Omega,\ t \in \mathbb{R},$$