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Dynamics of a Predator-Prey Model with Allee Effect and Herd Behavior

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Abstract This paper deals with dynamics of a predator-prey model with Allee effect and herd behavior. We first study the stability of non-negative constant solutions for such system. We also establish the existence of Hopf bifurcation solutions for such predator-prey model. The stability and bifurcation direction of Hopf bifurcation solution in the case of spatial homogeneity are further discussed. At the same time, several examples are given by MATLAB. Finally, the numerical simulations of the system are carried out through MATLAB, which intuitively verifies and supplements the theoretical analysis results.

Keywords Allee effect, herd behavior, stability analysis, Hopf bifurcation, numerical simulations

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1. Introduction

Biological mathematics is an interdisciplinary subject integrating mathematics and biology. Mathematical ecology is one of the most widely studied branches of mathematical biology. In particular, the ecological mathematical models in mathematical ecology are to reasonably establish the population dynamics models according to the relationship between population and population or between population and environment. So far, experts and scholars have made many important achievements in the research of ecological mathematical models.

In the ecological mathematical model, there is a model to describe the survival mode of two populations: population A depends on natural resources and population B feeds on population A, where population A is called prey and population B is called predator in ecology. Together, they form a predator-prey model. Predator-prey model is a kind of vital population model in ecological mathematical model, which has been widely studied by scholars [1–13]. The dynamic behavior of the predator-prey system is one of the focuses of mathematical ecology. There are intraspecific cooperation and competition in predator-prey system. Allee effect refers to the positive correlation between individual adaptability and population size or density [14]. In 2002, Petrovskii et al. [8] showed that Allee effect makes patchy invasion in predator-prey system possible. In 2012, Sen et al. [10] studied the bifurcation analysis of a ratio-dependent predator-prey model with Allee effect. In

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2016, Pal et al. [1] studied algae-herbivore interactions with Allee effect and chemical defense. In 2019, Xu et al. [15] cosidered the dynamics of species population for a regime switching model with environmental noises and additive Allee effect. In 2023, Kumbhakar et al. [16] investigated the dynamics of a predator-prey model with strong Allee effect in prey and a new kind of functional response by considering spatially grouped predators. At the same time, Xie and Zhang [17] studied the dynamic behaviors of a fractional order predator-prey system with Allee effect, fear effect and shelter effect.

In addition to intraspecific relationships, there are also interspecific relationships in predator-prey systems, most of which are predator-prey relationships or competition relationships between predators. Relatively few experimental or theoretical studies have explored the impact of the defensive behavior of prey against predators on the dynamics of the interaction between predators and prey. With the deepening of research, scholars began to pay attention to the defensive behavior of prey against predators, among which herd behavior is one of the most widely studied defensive behaviors of prey against predators. Herd behavior refers to that when a group shows a specific collective behavior, the individuals in the group will have the similar social behavior. Ajraldi et al. [18] expressed this idea, and proposed a predator-prey model with a single square root functional response function. Braza [3] discussed a predator-prey system with a modified Lotka-Volterra interaction term, in which the Lotka-Volterra interaction term is proportional to the square root of the prey population. In addition, they also compared the above conclusion with the dynamics of the predator-prey system with the classic Lotka-Volterra interaction term. Gimmelli et al. [6] studied a predator-prey system with herd behavior and predators carrying infectious diseases. In 2022, Brahim et al. [19] used a fractional-order model to show the effect of harvesting on a three-species predator-prey interaction in the case of prey herd behavior. Meanwhile, Shivam et al. [20] studied the temporal and spatiotemporal analysis of a prey-predator model with cooperative hunting among predators and herd behavior in prey. In 2023, Fordjour et al. [21] investigated a deterministic predator-prey model with prey herd behavior, mutual interference and the effect of fear.

At present, there are relatively few studies considering Allee effect and herd behavior in the predator-prey system at the same time [2, 7, 9, 12]. Particularly, in [12], Ye et al. studied the dynamics of the following predator-prey model with Allee effect and herd behavior in the spatially homogeneous situation

$$\begin{cases} \frac{dX}{dT} = rX(1-\frac{X}{K})(X-m) - \frac{\alpha\sqrt{X}Y}{1+T_h\alpha\sqrt{X}}, & T>0, \\ \frac{dY}{dT} = -\delta Y + \frac{c\alpha\sqrt{X}Y}{1+T_h\alpha\sqrt{X}}, & T>0, \\ X(0) = X_0 \geq 0,\ Y(0) = Y_0 \geq 0, \end{cases}$$

where X and Y represent the densities of prey and predator, respectively, α is the predator's search efficiency for prey, T_h is the average processing time of each prey, c is the conversion efficiency from prey to predator, δ is the natural mortality rate of predator, r is the intrinsic growth rate, K is the environmental capacity, m is the Allee effect threshold satisfying $-K < m < K$, $rX(1-\frac{X}{K})(X-m)$ is the Allee effect term and $\frac{\alpha\sqrt{X}Y}{1+T_h\alpha\sqrt{X}}$ is the herd behavior term. In order to better study the