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Exponential Stability of Positive Conformable BAM Neural Networks with Communication Delays

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Abstract In this paper, we consider a class of nonlinear differential equations with delays described by conformable fractional derivative. This type of differential equations can be used to describe dynamics of various practical models including biological and artificial neural networks with heterogeneous time-varying delays. By novel comparison techniques via fractional differential and integral inequalities, we prove under assumptions involving the order-preserving property of nonlinear vector fields that, with nonnegative initial states and inputs, the system state trajectories are always nonnegative for all time. This feature, called positivity, induces a special character, namely the monotonicity of the system. We then derive tractable conditions in terms of linear programming and prove, by utilizing the Brouwer's fixed point theorem and comparisons induced by the monotonicity, that the system possesses a unique positive equilibrium point which attracts exponentially all state trajectories. An application to the exponential stability of fractional linear time-delay systems is also discussed. Numerical examples with simulations are given to illustrate the theoretical results.

Keywords Conformable derivative, time-delay systems, BAM neural networks, positive equilibrium, M-matrix

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1. Introduction

Consider a class of nonlinear fractional differential delay equations of the form

$$ {}^{c}D^{\alpha}_{t_0}\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = -D_{\beta,\gamma}\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} + \begin{pmatrix} Af(y(t)) \\ Cg(x(t)) \end{pmatrix} + \begin{pmatrix} Bf(y_\sigma(t)) \\ Dg(x_\tau(t)) \end{pmatrix} + \begin{pmatrix} I \\ J \end{pmatrix}, \tag{1.1} $$

where ${}^{c}D^{\alpha}_{t_0}$ represents the conformable fractional derivative (CFD). More details on CFD and the description of system (1.1) will be presented in the next section. System (1.1) can be used to describe dynamics of various practical models such as fractional Hopfield-type neural networks or bidirectional associative memory (BAM)

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neural networks [1,2]. System (1.1) also encompasses important classes of time-delay systems like fractional linear systems with delays.

The theory of fractional calculus is one of the most active research areas in the past few years due to its demonstrated applications in numerous practical models such as data analysis, intelligent control, associative memory or optimization [3]. It has been recognized that functional calculus and fractional differential equations (FrDEs) can more adequately describe many physical phenomena compared to integer-order models. Several different approaches to fundamental concepts like fractional derivatives (FrDs) or fractional integrals (FrIs) have been developed in the senses of the Riemann-Liouville, Caputo or Grünwald-Letnikov [3, 4]. For example, the approach of Riemann-Liouville is constructed based on iterating the integral operator n times combining with the Cauchy formula, while the approach of Grünwald-Letnikov is based on iterating the derivative n times combining with the use of Gamma function in the binomial coefficients. The concepts of FrDs formulated in this direction are quite complicated in applications and have some common drawbacks. For instance, some basic properties of usual derivatives like product rule or chain rule are not preserved for FrDs. In addition to this, the monotonicity of a function f typically cannot be determined by FrDs of f in certain meanings.

To overcome some drawbacks of existing FrDs, the authors of [5] proposed a new well-behaved simple derivative called the conformable fractional derivative (CFD). Basic results in calculus of functions subject to CFD were also developed in [6, 7]. Recently, conformable fractional-order systems have also attracted considerable research attention and a number of interesting results involving various aspects of analysis and control of dynamical systems described by conformable fractional-order differential equations with or without delays have been published. For a few references, we refer the reader to recent works [8–15].

Positive systems form a particular class of dynamical systems, whose states and outputs starting from nonnegative inputs are always nonnegative. This type of systems is widely used to describe dynamics of many practical models in a variety of disciplines from biology, ecology and epidemiology, chemistry, pharmacokinetics to air traffic flow networks, control engineering, telecommunication and chemical-physical processes [16]. In the past few decades, the theory of positive systems has been intensively studied for various kinds of linear systems and nonlinear systems in integer-order models (see, e.g., [17–19] and the references therein). However, this area is still considerably less well-developed for fractional nonlinear systems, in particular, for models arising in artificial and biological neural networks.

The research topic of fractional differential equations and fractional neural networks (FrNNs) has received growing attention in recent years [20–22]. Some important issues in analysis such as stability, passivity, disspativity or identification and H_∞ control have also been extensively studied and developed for neural network models with delays (see, e.g., [2, 20, 23–26] and the references therein). However, there are only a few works concerning stability of conformable FrDEs. In particular, the positivity characterization and the existence, uniqueness and exponential stability of conformable delay systems in the form of (1.1) have not been studied. This motivates our present study.

In this paper, we consider a class of nonlinear delay differential equations described by conformable fractional derivative as presented in Eq. (1.1). This type of differential equations can be used to describe dynamics of various models in practice. By novel comparison techniques via fractional differential and integral