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On Nonlocal Neutral Stochastic Integro Differential Equations with Impulsive Random

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Abstract In this work, we discuss the existence and continuous dependence on initial data of solutions for non-local random impulsive neutral stochastic integrodifferential delayed equations. First, we prove the existence of mild solutions to the equations by using Krasnoselskii's-Schaefer type fixed point theorem. Next, we prove the continuous dependence on initial data results under the Lipschitz condition on a bounded and closed interval. Finally, we propose an example to validate the obtained results.

Keywords Existence, continuous dependence, random impulsive, integro differential equations, Krasnoselskii's-Schaefer type fixed point theorem

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1. Introduction

The theory of neutral differential equations (NDES) in Banach spaces has been studied by several authors [7], [9], [10], [12]. A neutral functional differential equation is one that includes both the current state of the system and the implied derivatives of the past history or functionals of the past history. When dealing with problems involving electric networks with lossless transmission lines, NDEs are required. Such networks first appeared, for instance, in high-speed computers where switching circuits were connected by lossless transmission lines. The problem's importance stems from the fact that it differs from the traditional initial condition in that it is more general and has a finer influence. The presence of solutions for neutral functional integrodifferential equations (IDEs) in Banach spaces was investigated by the authors [11], [13], [30]. The authors [5], [14], [29] proved that several classes of IDEs in abstract spaces exist as well as controllability results.

The impulses are either deterministic or random in that they occur at predetermined times or at random periods. There are numerous articles that examine the qualitative characteristics of fixed-type impulses [3], [4], [8], [15], [21], [24], [25], [26], [31]. but few that examine random-type impulses. The first random impulsive ordinary differential equations (ODEs) were presented by Wu and Meng [16], who also investigated the boundedness of these models' solutions using Liapunov's direct function. Some qualitative characteristics of differential equations (DEs) with random impulses have been researched by Wu et al. [17], [18], [22]. Anguraj et al. [2] established the stability of random impulsive stochastic functional DEs

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driven by Poisson jumps with finite delays by using Banach fixed point theorem. Li et al. [19] investigated the existence and Hyers-Ulam (HU) stability of mild solutions for random impulsive stochastic functional ODEs using Krasnoselskii's fixed point theorem. In Baleanu et al. [6] the existence, uniqueness, and HU (Hyers-Ulam) stability of random impulsive stochastic IDEs with nonlocal conditions have been investigated. By using Banach fixed point theorem,

$$\begin{aligned} d(z(t)) &= \big[\mathfrak{A}z(t) + f(t,z_t) + \int_0^t k(t-s)z(s)ds\big]dt + g(t,z_t)dW(t),\ t \geq t_0,\ t \neq \sigma_q, \\ z(\sigma_q) &= \ b_q(\delta_q)z(\sigma_q^-),\ q = 1,2,..., \\ z_0 &= z_{t_0} + r(z). \end{aligned}$$

Motivated by the above works, this paper aims to fill this gap by investigating the existence and continuous dependence on initial data of solutions of non-local random impulsive neutral stochastic integrodifferential equations (NRINSIDEs) with finite delays. By using Krasnoselskii's-Schaefer type fixed point theorem.

We consider the following NRINSIDEs with finite delays of the type

$$d[z(t) + h(t,z_t)] = \big[f(t,z_t) + \int_0^t k(t,s,z_s)ds\big]dt + g(t,z_t)dW(t), \tag{1.1}$$

$$z(\sigma_q) = b_q(\delta_q)z(\sigma_q^-),\ q = 1,2,..., \tag{1.2}$$

$$z_{t_0} + r(z) = z_0 = \sigma = \{\sigma(\theta) : -\delta \leq \theta \leq 0\}, \tag{1.3}$$

where δ_q is random variable defined from Ω to $\mathcal{D}_q \stackrel{def}{=} (0, d_q)$ for $q = 1,2,...,\ 0 < d_q < \infty$. Moreover, suppose that $\delta_\imath$ and $\delta_\jmath$ are independent of each other as $\imath \neq \jmath$ for $\imath, \jmath = 1,2....$ Here $f : [t_0, \mathcal{T}] \times \mathfrak{C} \to \mathbb{R}^d$, $h : [t_0, \mathcal{T}] \times \mathfrak{C} \to \mathbb{R}^d$, $g : [t_0, \mathcal{T}] \times \mathfrak{C} \times \to \mathbb{R}^{d \times m}$, $k : [t_0, \mathcal{T}] \times [t_0, \mathcal{T}] \times \mathfrak{C} \to \mathbb{R}^d$, $r : \mathfrak{C} \to \mathfrak{C}$ and $b_q : \mathcal{D}_q \to \mathbb{R}^{d \times d}$ are Borel measurable functions, and z_t is $\mathbb{R}^d$-valued stochastic process such that

$$z_t = \{z(t+\theta) : -\delta \leq \theta \leq 0\},\ z_t \in \mathbb{R}^d.$$

We assume that $\sigma_0 = t_0$ and $\sigma_q = \sigma_{q-1} + \tau_q$ for $q = 1,2,....$ Obviously, $\{\sigma_q\}$ is a process with independent increments. The impulsive moments σ_q from a strictly increasing sequence, i.e $\sigma = \sigma_0 < \sigma_1 < \sigma_2 < ... < \lim_{q\to\infty} \sigma_q = \infty$, and $z(\sigma_q^-) = \lim_{t\to\sigma_q - 0} z(t)$. Denote by $\{\mathbb{G}(t), t \geq 0\}$ the simple counting process generated by $\{\sigma_q\}$, let $\{\mathbb{K}(t), t \geq 0\}$ be a given m-dimensional Wiener process, and denote $\mathfrak{F}_t^{(1)}$ the σ-algebra generated by $\{\mathbb{G}(t), t \geq 0\}$. Denote $\mathfrak{F}_t^{(2)}$ the σ-algebra generated by $\{\mathbb{K}(s), s \leq t\}$.

For considering the main Eq. (1.1), we have

$$d(x(0)) = 0.$$

Here, extra conditions have to be imposed to guarantee the existence of a solution, so we refer to Lemmas 3.1, 3.2 and 4.1 in [27], and also, see Lemma 3.4 in [28].

Highlights:

1. This work extends the work of A. Vinodkumar. [6].
2. Time delay of NRINSIDEs is taken care of by the prescribed phase space $\mathcal{B}$.

The structure of this article is as follows. In section 2, we mention some concepts and principles. Section 3 discusses the existence of solutions for NRINSIDEs with