Journal of Nonlinear Modeling and Analysis
Volume 7, Number 1, February 2025, 1–19
https://journal.global-sci.org/jnma
DOI:10.12150/jnma.2025.1

# Stability and Bifurcation Analysis for a Predator-Prey Model with Crowley-Martin Functional Response

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**Abstract** This paper mainly focuses on the Gauze-type predator-prey model with Crowley-Martin functional response. The local stability of the equilibria is investigated by analyzing the characteristic equation and using the Routh-Hurwitz criterion. Besides, dynamic behavior has been studied by using the center manifold theorem and normal form theory. Finally, several numerical simulations not only verify the theoretical results of Hopf bifurcation but also display more interesting dynamical properties of the model.

**Keywords** Predator-prey system, Crowley-Martin functional response, Hopf bifurcation, center manifold theorem, normal form theory

**MSC(2010)** 34C23, 37C75, 37N25.

## 1. Introduction

Mathematical models in ecology provides us with information on the dynamical behavior of the ecological system. The interactions between predator and prey in nature are largely responsible for the rich biodiversity of complex ecosystems. So, it is necessary to analyse the predator-prey relationship qualitatively and quantitatively. The most paramount and critical term in the predator-prey model is the functional response function, which reflects the relationship between the predator population density and the prey population density. Most functional response functions depend either on the predator or the prey, such as classic Holling types functional response which were regarded as "prey-dependent" [5]. However, that is not entirely consistent with the actual situation in nature.

Let $x(t)$ and $y(t)$ denote densities of the prey and predators at time $t$, respectively. The classical Gauze-type predator-prey system without considering the spatial effect takes the following form [6]:

$$\begin{cases} x'(t) = xg(x) - yP(x,y), \\ y'(t) = \eta y P(x,y) - \mu y, \end{cases} \tag{1.1}$$

where $g(x) = r(1 - x/K)$, $r$ is the growth rate of prey and $K$ is called the carrying

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capacity of prey. $P(x, y)$ indicates the consumption rate of prey by a predator or the functional response of the predator.

Skalski and Gilliam [20] separately studied three functional response functions that reveal the interaction between predator and prey, and obtained rich statistical evidence from 19 predator-prey ecosystems. Research indicates that the Crowley-Martin functional response performed better in the three. Since this functional response recognizes the rich and biologically reasonable dynamics [1, 21], it is necessary for us to study it.

The Cowley-Martin functional response considered in this article takes the following form

$$P(x, y) = \frac{mx}{(a + bx)(a + cy)}, \tag{1.2}$$

where $m$, $a$, $b$, $c$ are positive constants that stand for the effects of consumption rate, saturation constant, handling time and the magnitude of interference among predators, respectively.

Over the past few decades, more and more information about the predator-prey system with Crowley-Martin functional response has been available [10, 14, 23, 24]. Zhou [26] presented qualitative analysis of the predator-prey model for Crowley-Martin functional responses. Li et al. [9] examined a predator-prey model with Crowley-Martin functional response for positive periodic solutions. Maiti et al. [12, 13] considered a prey-predator model which divides the scope of biological activity into two areas. They analysed the direction of Hopf bifurcation and the stability of the bifurcation periodic solution, and used the $V$ function to judge the global stability of the time-delay system. Upadhyay and Naji [24] studied a three food chain system with Crowley-Martin functional response and got local and global stability for non-negative equilibrium, conditions for the persistence of the system and bifurcation diagrams. Shi et al. [19] considered a predator-prey model with Crowley-Martin function, they considered the locally asymptotic stability of nonnegative equilibria and obtained sufficient conditions for the global stability of the positive equilibrium. Saha [17] studied a predator-predator model with a small parameter $\epsilon$ which is a slow-fast system that causes singular perturbation problem in mathematics. Besides, Santra etc [18] investigated the dynamics of a discrete predator-prey model, focusing on the Neimark-Sacker bifurcation. Another important bifurcation of the discrete differential equation is the Flip bifurcation, so we will also study the Flip bifurcation corresponding to the discrete system.

In this paper, we consider a predator-prey system model with a small parameter $\epsilon$, which is, the significant difference in population density between the predator and prey, such as spruce-budworm system [2, 11, 15, 22]. The main contributions of this work are summarized in the following.

(1) We carried out the stability analysis of the system, and determined the bifurcation parameter of the system as $\delta$ through mathematical analysis of the system model.

(2) We calculated the critical value $\delta_0$ and verified that the system occurs Hopf bifurcation when $\delta$ passes $\delta_0$ from right to left. We also analysed the direction of Hopf bifurcation and the stability of the bifurcation periodic solution.

(3) We also gave conditions for the corresponding discrete system to have a flip bifurcation.

(4) The example given in the numerical simulation is subcritical Hopf bifurcation, the bifurcating periodic solutions are stable and their period increases with