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Chaos Control and Behavior Analysis of a Discrete-Time Dynamical System with Competitive Effect*

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Abstract This paper is about a class of discrete-time dynamical systems with competitive effects. The local stability of the positive equilibrium point of the system and the conditions for the existence of flip bifurcation and Neimark-Sacker bifurcation are discussed by using the center manifold theorem and bifurcation theory. In addition, the direction of the flip bifurcation and Neimark-Sacker bifurcation is given. Furthermore, a feedback control strategy is employed to control bifurcation and chaos in the system. Finally, flip bifurcation, Neimark-Sacker bifurcation and chaos control strategy are verified with the help of numerical simulations.

Keywords Predator-prey system, flip bifurcation, Neimark-Sacker bifurcation, competitive effect, chaos

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1. Introduction

In nature, no species exists alone, but is closely related to and interdependent with other species. There is not only cooperation but also competition between the various groups. If two populations live together in the same area and compete for limited resources, space and other supplies, both sides of the competition are inhibited. In most cases, only one party benefits while the other party is eliminated, and one party replaces the other. This suggests that two different populations with the same needs cannot live permanently in the same environment. Interspecific competition is one of the most common interactions in predator-prey system. Berryman [1] studied the origin and evolution of predator-prey. Many researchers studied deterministic mathematical models in ecology and the dynamic behavior of prey-predator systems [2–11] . Furthermore, some authors studied the conditions, complexity and stability of spatial pattern formation in prey-predator systems [12–14].

Numerous studies have demonstrated that, for small populations, the discrete-time system is more appropriate than the continuous system. This has been effectively explored and explained in references [15–19]. In addition, Cheng et al. [20] conducted a study on a discrete-time prey-predator system with ratio-dependent

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and Allee effect, and found that the model with logistic growth function had similar bifurcation structures. Recent studies have demonstrated that the discrete-time prey-predator systems have more colorful dynamic behaviors, including bifurcation and chaos, than continuous systems. These dynamic behaviors between populations have been explored and analyzed through numerical simulations (references [21–26]). In [27–30], researchers not only studied the bifurcation phenomenon and dynamic behavior in the prey-predator systems, but also discussed the chaos control strategy for the chaos phenomenon at the unstable equilibrium point. Sarwardi et al. [31] studied a competitive prey-predator system with a prey refuge and obtained the relevant dynamic behaviors.

Interspecific competition is mainly reflected in the competition of different species for resources in the ecosystem (such as food, living space, etc.); interspecific competition is more common among closely related species. In fact, up to now, ecologists still have little understanding of the evolutionary significance of interspecific competition. On the one hand, researchers pay more attention to the form and process of competition, and on the other hand, it takes a long time for evolution to reflect the evolutionary significance of a biological event.

In order to study the effects of competition on populations, in this paper we consider the predator-prey system:

$$\begin{cases} u_{n+1} = u_n + \mu[r_1 u_n(1 - \frac{u_n}{K_1}) - \frac{b u_n v_n}{u_n + v_n} - \varepsilon_1 u_n^2], \\ v_{n+1} = v_n + \mu[r_2 v_n(1 - \frac{v_n}{K_2}) + \frac{a b u_n v_n}{u_n + v_n} - \varepsilon_2 v_n^2], \end{cases} \tag{1.1}$$

where $r_1, r_2, K_1, K_2, \varepsilon_1, \varepsilon_2, a$ and b are greater than zero, r_1 and r_2 are the intrinsic growth rates of the prey u and predator v populations, respectively. b indicates the ability of the predator to consume the prey. ε_1 and ε_2 denote the competition among individuals (i.e., intraspecies interaction) of prey and predator species due to resources. a denotes the conversion rate of the predator to the prey and K_1, K_2 denote environmental carrying capacity of the prey and predator in a particular habitat. And μ expresses the integral step length.

This paper is organized as belows. In Section 2, the existence and stability of the system at different equilibrium points are discussed. In Section 3, we discuss the specific conditions for the existence of Neimark-Sacker bifurcation and flip bifurcation. In Section 4, chaos is controlled to an unstable equilibrium point by the feedback control method. In Section 5, we carry out numerical simulations. Finally, we conclude with a brief summary in the last section.

2. Qualitative study of system

In this section, we will investigate the existence and stability of fixed points in the system. To determine the equilibrium points of equation (1.1), we solve the following set of equations:

$$\begin{cases} u = u + \mu[r_1 u(1 - \frac{u}{K_1}) - \frac{buv}{u+v} - \varepsilon_1 u^2], \\ v = v + \mu[r_2 v(1 - \frac{v}{K_2}) + \frac{abuv}{u+v} - \varepsilon_2 v^2]. \end{cases}$$

By calculation, the following results can be gained directly.